Section 6.14 #6

Calculate A^n and express A^3 in terms of I, A, and A^2

Used Corollary 6.14 (page 210)

A = [0 1 1]
[0 1 1]
[0 0 0]

$f(\lambda) = \det(\lambda I - A)$

$A^3 = AA^2$

$\lambda I - A$ = [λ 0 0] – [0 1 1] = [λ -1 -1]
[0 λ 0] [0 1 1] [0 (λ-1) -1]
[0 0 λ] [0 0 0] [0 0 λ]

det([λ -1 -1] [0 (λ-1) -1] [0 0 λ]) $= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32} - a_{13}a_{22}a_{31}$

$= \lambda^2(\lambda-1) + (-1)^2*0 + \lambda*0 - (-1)\lambda*0 - \lambda(-1)*0 - (-1)(\lambda-1)*0$

$= \lambda^2(\lambda-1) = \lambda^3 - \lambda^2 + 0*\lambda + 0$

$c_0 = 1, c_1 = 0, c_2 = -1, c_3 = 1$

$A^3 = c_2*A^2 + c_1*A + c_0*I$

$\mathbf{A^3 = 1*A^2 + 0*A + 0*I}$

We know…

A*A = [0 1 1] = A^2 = A
[0 1 1]
[0 0 0]

If n is an even number then $A^n = (A^2)^k$

$A^n = A^2 * A^2 * A^2 * A^2 \ldots$

$A^n = A * A * A * A \ldots$

$A^n = A^2 * A^2 \ldots$

$A^n = A$

If n is an odd number then $A^n = (A^2)^k A$

$A^n = A^2 * A^2 * A^2 * A^2 * \ldots * A$

$A^n = A * A * A * A * \ldots * A$

$A^n = A^2 * A^2 * \ldots * A$

$A^n = A * A = A$

If k is not a multiple of 2, then an extra A^2 will be produced leaving.

$A^n = A * A^2$

$A^n = A * A = A$

So..

A^n = [0 1 1] = A
[0 1 1]
[0 0 0]

Section 7.4 #5

Let V = $\mathbf{R}^3$ with all the usual dot product as inner product. Let **T** be a reflection in the xy plane; that is let T(i) = i, T(j) = j, T(k) = -k. Prove that T is symmetric.

If we can prove T is Hermitian, then it is symmetric. (by defn page 218)
T is Hermitian if…

(T(x), y) = (x, T(y)) for all x and y in V

Any element x and y in V can be written as a linear combination of i, j, and k

x = a*i + b*j + c*k	y = d*i + e*j + f*k
T(x) = T(a*i + b*j + c*k)	T(y) = T(d*i + e*j + f*k)
T(x) = aT(i) + bT(j) + cT(k)	T(y) = dT(i) + eT(j) +fT(k)
T(x) = a*i + b*j - c*k	T(y) = d*i + e*j - f*k

(T(x), y) = [a b –c]•[d e f] = ad + be –cf
(x, T(y)) = [a b c] •[d e –f] = ad + be – cf

(T(x), y) = (x, T(y)) so T is Hermitian and therefore symmetric!