Name:

Section:

Calculus 1 **EXAM 3** Fall, 2007

General Instructions:

You may use one, double-sided, 8.5X11 inch formula sheet. Put your NAME on it and turn it in with the exam.

You may not use any books, calculator, or electronic equipment.

You may not look at anyone else's paper.

We have an honor code. If you notice any dishonest behavior, you should let the TA or professor know as soon as possible.

Use only the scrap paper at the back of the exam.

If you have any questions, raise your hand.

Try not to spend a disproportionate amount of time on problems that are worth few points.

If you have time, CHECK ALL ANSWERS.

Part I: Upper and Lower Sums and Definite Integrals (no partial credit)

Put ONLY YOUR FINAL ANSWER in the box.

1. (4points) Compute the <u>upper sum</u> $U_f(P)$ for $f(x) = 3x^2$ on $[-1, 2]$ with partition $P = \{-1, -1/2, 0, 2\}$.

2. (4 points) Compute
$$\int_{-1}^{2} 3x^2 \, dx.$$

3. (4 points) Compute
$$\int_{0}^{\pi/4} 3\sec^2(x) \, dx.$$

4. (4 points) Compute the average value of $f(x) = |x|$ on $[-1, 2]$.

Part II: Indefinite Integrals and Antiderivatives (no partial credit)

Put ONLY YOUR FINAL ANSWER in the box.

1. (4 points) Find a function $F(x)$ such that $F'(x) = \sqrt{x}$ and $F(0) = 1$.

2. (4 points) Compute

$$\frac{d}{dx} \int_5^{x^3} \sqrt{5 + t^2} \, dt.$$

3. (4 points) Calculate the indefinite integral:

$$\int \sin^2(x) \cos(x) \, dx.$$

4. (4 points) Find the critical point(s) and at each critical point, determine whether F has a local max, local min, or neither.

$$F(x) = \int_2^x \frac{4 - t}{1 + 2t^2} \, dt.$$

Part IV: Long answer (show all work). Answer BOTH of the following TWO questions. **Draw a BOX around YOUR FINAL ANSWER.**

1. Consider the region bounded by the graph of $y = x^2$, $y = 0$, $x = 1$, and $x = 2$.

   (a) (2 points) Find the area.

   (b) (3 points) Find the centroid.

   (c) (2 points) Use Pappus' Theorem to find the volume of the solid generated by revolving this region about the y-axis.

   (d) (3 points) Use ANOTHER METHOD to find the volume. Show all work.

2. Consider

$$f(x) = \frac{x}{(x+3)^2}.$$

(a) (1 point) What is the domain of f?

(b) (1 point) Find all x-intercepts of f.

(c) (1 point) Find all vertical asymptotes of f.

(d) (1 point) Find all horizontal asymptotes of f.

(e) (1 point) Find all critical points of f and classify (local max, local min, neither).

(f) (1 point) Where is f concave up? Where is f concave down?

(g) (1 point) Name all inflection points of f.

(h) (3 points) Using your answers to the previous parts, sketch the graph of f. Label your axes and a couple of representative points.