MATH 2401, PRACTICE TEST 1

1 Let $\mathbf{u}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t\mathbf{k}$, $\mathbf{v}(t) = \cos t \mathbf{i} - \sin t \mathbf{j} + t\mathbf{k}$ and $f(t) = e^{2t}$. Find

$$\frac{d}{dt}(\mathbf{u}\cdot\mathbf{v}) \qquad \frac{d}{dt}f(t)\mathbf{u}(t)$$

2 Find the length of the curve given by:

$$\mathbf{r}(t) = 6t\,\mathbf{i} + 3\sqrt{2}t^2\,\mathbf{j} + 2t^3\,\mathbf{k}$$

for t from 0 to 1.

- 3 The position of a particle is given by $\mathbf{r}(t) = 5t\mathbf{i} + 3t\mathbf{j} + (-2t^2 + 3t)\mathbf{k}$. When is the speed a minimum?
- 4 Find the curvature of the curve given by $\mathbf{r}(t) = 3\cos 2t \mathbf{i} + 3\sin 2t \mathbf{j} + t\mathbf{k}$.
- 5 Find the domain and range of the function $f(x, y, z) = \sqrt{x^2 + y^2 1} \ln z$ and write the equation of the level surface that contains the point $(1, 2, e^2)$.
- 6 Find the following limit or show that it does not exists:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$$
$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{\sqrt{x^7 + y^7}}$$

7 Compute

$$\frac{\partial^2}{\partial_x \partial_y} (y \cos y + xy \sin xy)$$