Chapter 6.2

Ex n. 3:

From Chebyshev Inequality we have

$$\mathbb{P}(|X-10|\geq 3) \leq \frac{Var(X)}{9}$$

but

$$\mathbb{P}(|X-10|\geq 3) = \mathbb{P}(X\leq 7) + \mathbb{P}(X\geq 10) = 0.5$$

so That

$$Var(X) \geq 9/2$$

Ex n. 7

Let

$$Y = |X-\mu|^4$$

From Markov inequality we get

$$\mathbb{P}(Y\geq t) \leq \frac{\mathbb{E}(Y)}{t}$$

that is

$$\mathbb{P}(|X-\mu|^4\geq t) \leq \frac{\beta_4}{t}$$ or

$$\mathbb{P}(|X-\mu|\geq t) \leq \frac{\beta_4}{s^4}$$

Ex n. 15

We want to show that for every $\delta > 0$

$$\lim_{n \to \infty} \mathbb{P}(|g(z_n) - g(b)| \geq \delta) = 0$$

By continuity we have that $\forall \delta \; \exists \delta'$ such that

$$|z - b| < \delta' \Rightarrow |g(z) - g(b)| < \delta$$

Thus

$$\mathbb{P}(|g(z_n) - g(b)| \geq \delta) \leq \mathbb{P}(|z_n - b| < \delta')$$

Since by assumption:

$$\lim_{n \to \infty} \mathbb{P}(|z_n - b| < \delta') = 0$$

The thesis follows.

Chapter 6.3

n. 4

We have that

$$\frac{\sqrt{n}}{3}(\bar{X}_n - \mu) = Z$$

is approximately $N(0,1)$. Thus

$$\mathbb{P}(|\bar{X}_n - \mu| < 0.3) = \mathbb{P}(|Z| < 0.1\sqrt{n}) \simeq$$

$$1 - 2\Phi(-0.1\sqrt{n})$$

and we need

$$\Phi(-0.1\sqrt{n}) \leq 0.025$$

Since

$$\Phi(-1.96) = 0.025$$

we get

$$0.1\sqrt{n} \geq 1.96$$

or

$$n \geq (19.6)^2 \simeq 400$$

Ex. n. 10

a) From eq. (6.2.4) we get

$$\mathbb{P}\left(|\overline{X}-\mu| \geq \frac{\sigma}{4}\right) \leq \frac{16}{n}$$

so that we need

$$\frac{16}{n} \leq 0.01 \qquad \text{or}$$

$$n \geq 1600$$

b) Let

$$Z = \frac{\sqrt{n}}{\sigma}(\overline{X}_n - \mu)$$

Then Z is approximately $N(0,1)$ so that

$$\mathbb{P}\left(|\overline{X}_n - \mu| \leq \frac{\sigma}{4}\right) = \mathbb{P}\left(|Z| \leq \frac{\sqrt{n}}{4}\right) \simeq$$

$$\simeq 1 - 2\Phi\left(-\frac{\sqrt{n}}{4}\right)$$

Since $\Phi(-2.58) = 0.005$ we need

$$\frac{\sqrt{n}}{4} \geq 2.58 \qquad \text{or}$$

$$n \geq (4 \cdot 2.58)^2 \simeq 107.$$

6.4 n 2

We have

$$\mathbb{E}(X) = 0.3 \cdot 15 = 4.5$$

$$Var(X) = 0.3 \cdot 0.7 \cdot 15 = 3.15$$

a) From The ~~ced~~ CLT we get

$$\mathbb{P}(X=4) \simeq \mathbb{P}\left(3.5 \leq \sqrt{3.15}\, Z + 4.5 \leq 4.5\right)$$

where $Z \simeq \mathcal{N}(0,1)$. Thus

$$\mathbb{P}(X=4) \simeq \Phi(0) - \Phi(-0.56) = 0.2123$$

b) For The exact value we get

$$\mathbb{P}(X=4) = 0.2186$$