Exercise 8.4 n. 3

Since

$$\mathbb{E}(X_i) = 0 \qquad var(X_i) = 1$$

we have

$$\frac{1}{2}(X_1 + X_2) \sim N(0, 1)$$

and

$$X_3^2 + X_4^2 + X_5^2 \sim \chi_3^2$$

Moreover

$$X_1 + X_2 \perp\!\!\!\perp X_3^2 + X_4^2 + X_5^2$$

so that

$$c = \frac{1}{2}$$

Exercise 8.5 n 1

We have

$$\mathbb{P}\left(\bar{X}_n - \Phi^{-1}\left(\frac{1+\gamma}{2}\right)\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X}_n + \Phi^{-1}\left(\frac{1+\gamma}{2}\right)\frac{\sigma}{\sqrt{n}}\right) =$$

$$\mathbb{P}\left(-\Phi^{-1}\left(\frac{1+\gamma}{2}\right) \le \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \le \Phi^{-1}\left(\frac{1+\gamma}{2}\right)\right) =$$

$$= \mathbb{P}\left(Z \le \Phi^{-1}\left(\frac{1+\gamma}{2}\right)\right) - \mathbb{P}\left(Z \le -\Phi^{-1}\left(\frac{1+\gamma}{2}\right)\right)$$

$$= \frac{1+\gamma}{2} - \left(1 - \frac{1+\gamma}{2}\right) = \gamma$$

n. 4

The coefficient 0.95 C.I. for μ

is

$$\bar{X} - 1.96\,\sigma/\sqrt{N} \le \mu \le \bar{X} + 1.96\,\sigma/\sqrt{N}$$

we thus want

$$2 \cdot 1.96 \cdot \frac{\sigma}{\sqrt{N}} \le 0.01\,\sigma$$

or

$$N \ge \left(\frac{2 \cdot 1.96}{0.01}\right)^2 \approx 400^2$$

n. 7

Using The t Confidence interval we need

$$\bar{x} = 156.85$$

$$\hat{\sigma}' = 22.64$$

and $T_{19}^{-1}(0.95) = 1.729$.

We get The interval

$$\bar{x} - T_{19}^{-1}(0.95)\frac{\hat{\sigma}'}{\sqrt{20}} \leq \mu \leq$$

$$\bar{x} + T_{19}^{-1}(0.95)\frac{\hat{\sigma}'}{\sqrt{20}}$$

or

$$148.1 \leq \mu \leq 165.6$$