

Use only your textbook. No laptop or wireless devices allowed. Write clearly.

Name: \_\_\_\_\_

Question:	1	2	3	4	Total
Points:	40	20	20	20	100
Score:					

Question 1 ..... 40 point

The water from a new natural source is bottled but must pass health inspection to be approved. In particular the bottling company must check for the average content of a particular chemical A in the bottles. The following data  $x_i$  are the result of a random sample of size  $N = 50$  of the content of A in mg in the new bottles. They are ordered in increasing order.

2.02	2.04	2.05	2.12	2.17	2.25	2.26	2.33	2.36	2.37
2.39	2.46	2.49	2.49	2.58	2.64	2.71	2.76	2.79	2.84
2.93	2.94	3.06	3.07	3.09	3.09	3.11	3.26	3.29	3.32
3.34	3.37	3.43	3.45	3.46	3.47	3.51	3.51	3.55	3.64
3.64	3.70	3.77	3.84	3.85	3.85	3.86	3.87	3.90	3.98

You know that

$$\sum_{i=1}^{50} x_i = 152.27 \quad \sum_{i=1}^{50} x_i^2 = 481.45.$$

(a) (10 points) Compute the sample average and standard deviation.

(b) (10 points) Compute the median and fourth spread.

(c) (10 points) Draw a box plot for the data.

(d) (10 points) Choose a reasonable number of classes and draw an histogram for the data, using the densities. Show your computations.

Question 2 ..... 20 point

A baker produces every morning an amount of bread equal to the orders for that day he received the evening before. Call  $X$  this amount expressed in lbs and assume that it is an exponential r.v. with  $E(X) = m$ . The price of the bread is \$1/lb, that is he will get \$ $x$  if he sells  $x$  lbs of bread. Due to economy of scale, the cost  $c(x)$  of producing  $x$  lbs of bread is  $c(x) = \sqrt{x}$ . Compute the expected profit of the baker. Find the value of  $m$  for which the baker break even, that is his expected profit is \$0. (**Hint:** Call  $p(x)$  the profit of the baker when he receive orders for  $x$  lbs of bread. You have to compute  $E(p(X))$ . See last page for useful formulas.)

Question 3 ..... 20 point

You want to use a set of wires to suspend a weight for the ceiling of your workshop.

After many tests, you reach the conclusion that each wire used has a breaking point given by a random variable  $X$  with  $E(X) = 10\text{kg}$  and  $\sigma_X = 2\text{kg}$ .

You must suspend a weight of 500kg so you decide to use a large number of wires to suspend it. Suppose that the breaking point of a set of wires used together is the sum of the breaking points of each of them and that each wire is independent from the others.

Compute the probability the the wires will not break when you use 50, 54 or 58 of them. (**Hint:** since 50 is large, you can use the CLT.)

Question 4 ..... 20 point

Two discrete r.v.  $X$  and  $Y$  can take the value  $-1, 0$  and  $1$ . The joint p.m.f of the two variables is given by

		Y		
		-1	0	1
X	-1	0.1	0.15	0.1
	0	0.15	0.0	0.15
	1	0.1	0.15	0.1

(a) (10 points) Compute the expected values  $E(X)$ ,  $E(Y)$ , variances  $V(X)$ ,  $V(Y)$  and the correlation coefficient  $\rho_{X,Y}$  of  $X$  and  $Y$ .

(b) (10 points) Are  $X$  and  $Y$  independent?

**Useful Facts**

- **A useful Integral:**

$$\int_0^{\infty} x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$$

- **C.L.T.:** Let  $X_i, i = 1, \dots, N$ , be  $N$  independent and identically distributed r.v. Then  $T_0 = \sum_{i=1}^N X_i$  is a Normal r.v.