Spring 04 — Name: ______________________
Math 3770 — Third Midterm — Bonetto

1) You play with a friend the following game: you flip a coin and if the result is Head you win 1\$ while if it is Tail you lose 1\$ (*i.e.* you win -1\$). Let X_i the r.v. that describes the ammount you win at the i-th coin flip. You play the game 2500 times. Let $S = \sum_{i=1}^{2500} X_1$ the r.v. describing the total amount you win after these 2500 plays. Assume that the coin is fair, *i.e.* assume the $P(X_i = 1) = P(X_i = -1) = 0.5$.

a) Write an approximate p.d.f. for S.
b) What is the approximate probability that at the end of the game you lost more than 20\$? And more than 100\$? (Use the table at the end of the test)

At the end of the game you lost 300\$. You suspect that your friend cheated you and the coin is not fair. Assume now that $P(X_i = 1) = p$ and $P(X_i = -1) = 1 - p$

c) Give an estimate on p based on the previous result.
d) Give a 99% CI for p. (Remeber that $x_{0.005} = 2.58$ and that $V(X_i) = 4p(1-p)$)
e) (**Bonus**) Do you think the coin was fair? Why?

2) After running a sample of size $n = 100$ an experimenter find the following values:

0.985	0.118	0.894	0.784	0.101	0.253	0.020	0.378	0.679	0.681
0.753	0.006	0.624	0.126	0.618	0.771	0.187	0.497	0.510	0.316
0.702	0.367	0.876	0.371	0.525	0.428	0.543	0.668	0.171	0.831
0.862	0.769	0.053	0.967	0.334	0.082	0.270	0.074	0.676	0.810
0.464	0.569	0.512	0.587	0.739	0.872	0.578	0.965	0.334	0.393
0.177	0.112	0.853	0.811	0.955	0.904	0.139	0.686	0.127	0.004
0.001	0.193	0.587	0.375	0.027	0.465	0.785	0.442	0.288	0.853
0.526	0.728	0.266	0.783	0.468	0.197	0.447	0.728	0.966	0.757
0.466	0.869	0.134	0.846	0.018	0.941	0.670	0.800	0.143	0.610
0.908	0.774	0.366	0.944	0.885	0.445	0.163	0.425	0.480	0.031

that ordered in increasing order are:

0.001	0.004	0.006	0.018	0.020	0.027	0.031	0.053	0.074	0.082
0.101	0.112	0.118	0.126	0.127	0.134	0.139	0.143	0.163	0.171
0.177	0.187	0.193	0.197	0.253	0.266	0.270	0.288	0.316	0.334
0.334	0.366	0.367	0.371	0.375	0.378	0.393	0.425	0.428	0.442
0.445	0.447	0.464	0.465	0.466	0.468	0.480	0.497	0.510	0.512
0.525	0.526	0.543	0.569	0.578	0.587	0.587	0.610	0.618	0.624
0.668	0.670	0.676	0.679	0.681	0.686	0.702	0.728	0.728	0.739
0.753	0.757	0.769	0.771	0.774	0.783	0.784	0.785	0.800	0.810
0.811	0.831	0.846	0.853	0.853	0.862	0.869	0.872	0.876	0.885
0.894	0.904	0.908	0.941	0.944	0.955	0.965	0.966	0.967	0.985

a) Compute the median $\tilde{x}$ and the fourth spread f_s of the sample.

He wants to visualize the result with an histogram.

b) How many classes should he use?
c) Write the boundaries and the value for the second and sixth class.

The experimenter wants to know the population average μ. He observes that

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{100} x_i = 0.513 \qquad s^2 = \frac{1}{n-1}\sum_{i=1}^{100}(x_i^2 - \bar{x})^2 = 0.089$$

d) Write a 95% confidence interval for μ.($z_{0.025} = 1.96$)

The experimenter needs to know μ with a confidence level of 95% and a precision of 0.01 (*i.e.* he wants a CI of size at most 0.02).

e) Is the sample he has large enough? If not, how large should the sample be?

3) For theoretical reason we expect that the level of the water in a given river varies from day to day and is distributed according to the following p.d.f.:

$$f(x) = \begin{cases} 0 & x < 0 \\ \\ xe^{-\lambda x} & x \geq 0 \end{cases}$$

Let X_i, $i = 1, \ldots, n$ be a random sample of size n for this problem, *i.e.* the p.d.f. of X_i is $f(x_i)$ and the X_i are independent.

a) Write the joint p.d.f. of the sample $F(x_1, \ldots, x_n, \lambda)$.
b) Derive a MLE for the parameter λ.
c) (**Bonus**) Is the above estimator unbaised?