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Let call Y the r.v. that counts the number of working bulbs. Than Y is distributed as a binomial of parameters 10, p, *i.e.* $Y \simeq \text{Bin}(10, p)$.

1) the requested probability is

$$P(Y = 10) = b(10; 10, p) = p^{10} = 0.904.$$

2)

$$P(Y = 4) = b(4; 10, p) = \binom{10}{4} p^4 (1-p)^6 = 2.1 \cdot 10^{-10}$$

3) he expected value is $E(Y) = 10p = 9.9$ and the variance is $V(X) = 10p(1-p) = 0.099$

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4) Observe that after a mounth each bulb is working or not working independently from the others so the X is a binomial variable of parameters 10 and p', where p' is the probability that a given bulb is still working after a month. This probability is given by the probability that it was working at the initial time time the probability that it did not break down during the month of use, *i.e.* $p' = p(1-q) = 0.99 \cdot 0.9 = 0.891$. So we have that $X \simeq \text{Bin}(10, 0.891)$ and the p.m.f. of $P(X = x) = b(x; 10, 0.891)$. In particular

$$P(X = 8) = b(8; 10, 0.891) = \binom{10}{8} 0.891^8 \cdot 0.109^2 = 0.212.$$

You can reach the same result calling Z the random variable that counts how many bulbs brake down in a mounts and obeserving that, if $Y = y$, than $Z \simeq \text{Bin}(y, 0.1)$. Than

$$\begin{aligned} P(X = 8) =& P(Y = 10)P(Z = 2) + P(Y = 9)P(Z = 1) + P(Y = 8)P(Z = 0) = \\ =& b(10; 10, 0.99)b(2; 10, 0.1) + b(9; 10, 0.99)b(1; 9, 0.1) + \\ +& b(8; 10, 0.99)b(0, 8, 0.1) \end{aligned}$$

that gives the same result as above.

5) The requested probability is $P(A|B')$. Using the product rule we get

$$P(A|B') = P(B'|A)\frac{P(A)}{P(B')} = 0.1\frac{0.99}{0.109} = 0.908.$$

This is true because $P(B'|A)$ is the probability that the bulb breaks down during use and $P(B') = 1 - P(B)$ was computed in point 4.

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6) The probability that the light will not go on is 1 minus the probability that all bulbs work. This was computed in point 1 so that the requested probability is $1 - 0.904 = 0.086$.

7) The light does not go on if at least one of the bulbs is broken so that the events {*the light does not go on*} $= \bigcup_{i=1}^{10} A'_i$. Clearly the event {*the first bulb is not working* } $= A'_1$. Thus

$$P\left(A'_1 \middle| \bigcup_{i=1}^{10} A'_i\right) = \frac{P\left(A'_1 \cap \left(\bigcup_{i=1}^{10} A'_i\right)\right)}{P\left(\bigcup_{i=1}^{10} A'_i\right)} = \frac{P(A'_1)}{P\left(\bigcup_{i=1}^{10} A'_i\right)} = \frac{0.01}{0.086} = 0.116.$$

This is true because $A'_1 \subset \bigcup_{i=1}^{10} A'_i$ so that $A'_1 \cap \left(\bigcup_{i=1}^{10} A'_i\right) = A'_1$ and $P\left(\bigcup_{i=1}^{10} A'_i\right)$ was computed in point 6.

8) The event {*only the first bulb does not work*} is given by $C = A'_1 \cap \left(\bigcup_{i=2}^{10} A_i\right)$. We again have that $C \subset \bigcup_{i=1}^{10} A'_i$ so that the requested porbability is

$$P\left(A'_1 \cap \left(\bigcap_{i=2}^{10} A_i\right) \middle| \bigcup_{i=1}^{10} A'_i\right) = \frac{P\left(A'_1 \cap \left(\bigcap_{i=2}^{10} A_i\right)\right)}{P\left(\bigcup_{i=1}^{10} A'_i\right)} = \frac{0.01 \cdot 0.99^9}{0.086} = 0.106.$$

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9) Clearly we have that $Y \simeq H(10, 20, 2)$. The light goes on in Room 1 if and only if there are 0 non working bulbs in Room 1 so that the requested probability is

$$h(0; 10, 20, 2) = \frac{\binom{18}{10}\binom{2}{0}}{\binom{20}{10}} = \frac{18!}{10! \cdot 8!} \cdot \frac{10! \cdot 10!}{20!} = \frac{10 \cdot 9}{20 \cdot 19} = 0.237$$

10) the probability requested is $P(Y = 1)$ because if there is only one non working bulb in Room 1 the other non working bulb has to be in Room 2. We have

$$P(Y = 1) = h(1; 10, 20, 2) = \frac{\binom{18}{9}\binom{2}{1}}{\binom{20}{10}} = 2 \cdot \frac{18!}{9! \cdot 9!} \cdot \frac{10! \cdot 10!}{20!} = \frac{2 \cdot 10 \cdot 10}{20 \cdot 19} = 0.526.$$

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11) Let again Y be the r.v. that counts the number of non working bulbs in Room 1. We have $Y \simeq H(10, 20, 5)$ so that:

$$P(Y=3) = h(3;10,20,5) = \frac{\binom{15}{7}\binom{5}{3}}{\binom{20}{10}} = \frac{15!}{7!\cdot 8!}\cdot\frac{5!}{3!\cdot 2!}\cdot\frac{10!\cdot 10!}{20!} =$$

$$= \frac{10\cdot 9\cdot 8\cdot 10\cdot 9\cdot 5\cdot 4}{2\cdot 20\cdot 19\cdot 18\cdot 17\cdot 16} = 0.348$$

while

$$P(Y=0) = h(2;10,20,5) = \frac{\binom{15}{8}\binom{5}{2}}{\binom{20}{10}} = \frac{15!}{8!\cdot 7!}\cdot\frac{5!}{2!\cdot 3!}\cdot\frac{10!\cdot 10!}{20!} = 0.348.$$

12) The requested probability is:

$$P(Y=0) = h(0;10,20,5) = \frac{\binom{15}{10}\binom{5}{0}}{\binom{20}{10}} = \frac{15!}{10!\cdot 5!}\cdot\frac{10!\cdot 10!}{20!} =$$

$$= \frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{20\cdot 19\cdot 18\cdot 17\cdot 16} = 0.0162$$