MATH 6337 HOMEWORK #3 Due date: September 18, 2007

Work the following problems and hand in your solutions. You may work together with other people in the class, but you must each write up your solutions independently. A subset of these will be selected for grading. Write LEGIBLY on the FRONT side of the page only, and STAPLE your pages together.

1. Show that if $\mu^*$ is an outer measure on $X$ and $A$, $B$ are $\mu^*$-measurable, then

$$\mu^*(A \cup B) + \mu^*(A \cap B) \ = \ \mu^*(A) + \mu^*(B).$$

2. Problem 1.4 #18. Let $\mathcal{A} \subseteq \mathcal{P}(X)$ be an algebra, $\mathcal{A}_\sigma$ the collection of countable unions of sets in $\mathcal{A}$, and $\mathcal{A}_{\sigma\delta}$ the collection of countable intersections of sets in $\mathcal{A}_\sigma$. Let $\mu_0$ be a premeasure on $\mathcal{A}$ and $\mu^*$ the induced outer measure.

a. For any $E \subseteq X$ and $\varepsilon > 0$ there exists $A \in \mathcal{A}_\sigma$ with $E \subseteq A$ and $\mu^*(A) \leq \mu^*(E) + \varepsilon$.

b. If $\mu^*(E) < \infty$, then $E$ is $\mu^*$-measurable if and only if there exists $B \in \mathcal{A}_{\sigma\delta}$ with $E \subseteq B$ and $\mu^*(B \setminus E) = 0$.

c. If $\mu_0$ is $\sigma$-finite, the restriction $\mu^*(E) < \infty$ in part b is superfluous.

3. Problem 1.4 #19. Let $\mu^*$ be an outer measure on $X$ induced from a finite premeasure $\mu_0$. If $E \subseteq X$, define the *inner measure* of $E$ to be $\mu_*(E) = \mu_0(X) - \mu^*(E^{\mathrm{C}})$. Then $E$ is $\mu^*$-measurable if and only if $\mu^*(E) = \mu_*(E)$.

Hint: Apply Problem 1.4 #18 part a to both $E$ and $E^{\mathrm{C}}$.