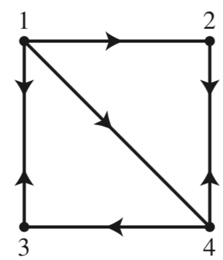
Supplemental problems: §5.6

1. Suppose the internet has four pages in the following manner. Arrows represent links from one page towards another. For example, page 1 links to page 4 but not vice versa.



a) Write the importance matrix and the Google matrix for this internet using damping constant $p = 0.15$. You don't need to simplify the Google matrix.

b) The steady-state vector for the Google matrix is (approximately)

$$\begin{pmatrix} 0.23 \\ 0.23 \\ 0.23 \\ 0.31 \end{pmatrix}.$$

What is the top-ranked page?

2. The companies X, Y, and Z fight for customers. This year, company X has 40 customers, Company Y has 15 customers, and Z has 20 customers. Each year, the following changes occur:

- X keeps 75% of its customers, while losing 15% to Y and 10% to Z.
- Y keeps 60% of its customers, while losing 5% to X and 35% to Z.
- Z keeps 65% of its customers, while losing 15% to X and 20% to Y.

Write a stochastic matrix A and a vector x so that Ax will give the number of customers for firms X, Y, and Z (respectively) after one year. You do not need to compute Ax.

3. Suppose p and q are real numbers on the open interval $(0, 1)$, and

$$A = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix}$$

(1) Is A a positive stochastic matrix? Why?
(2) Does A have unique steady state vector? Why?
(3) Without computation, give an eigenvalue of A.
(4) Compute the steady-state vector of A.

Supplemental problems: Chapter 6

1. True or false. If the statement is always true, answer true. Otherwise, answer false. Justify your answer.

a) Suppose $W = \mathrm{Span}\{w\}$ for some vector $w \neq 0$, and suppose v is a vector orthogonal to w. Then the orthogonal projection of v onto W is the zero vector.

b) Suppose W is a subspace of $\mathbf{R}^n$ and x is a vector in $\mathbf{R}^n$. If x is not in W, then $x - x_W$ is not zero.

c) Suppose W is a subspace of $\mathbf{R}^n$ and x is in both W and $W^\perp$. Then $x = 0$.

d) Suppose $\widehat{x}$ is a least squares solution to $Ax = b$. Then $\widehat{x}$ is the closest vector to b in the column space of A.

2. Let $W = \mathrm{Span}\{v_1, v_2\}$, where $v_1 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

a) Find the closest point w in W to $x = \begin{pmatrix} 0 \\ 14 \\ -4 \end{pmatrix}$.

b) Find the distance from w to $\begin{pmatrix} 0 \\ 14 \\ -4 \end{pmatrix}$.

c) Find the standard matrix for the orthogonal projection onto $\mathrm{Span}\{v_1\}$.

d) Find the standard matrix for the orthogonal projection onto W.

3. Find the least-squares line $y = Mx + B$ that approximates the data points

$$(-2, -11), \quad (0, -2), \quad (4, 2).$$