Math 1553 Worksheet §§3.5-4.3

1. True or false. Answer true if the statement is *always* true. Otherwise, answer false. If your answer is false, either give an example that shows it is false or (in the case of an incorrect formula) state the correct formula.

a) If A and B are $n \times n$ matrices and both are invertible, then the inverse of AB is $A^{-1}B^{-1}$.

b) If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in $\mathbf{R}^n$, then the solution is *unique* for each b in $\mathbf{R}^n$.

c) If A is an $n \times n$ matrix and the equation $Ax = b$ has at most one solution for each b in $\mathbf{R}^n$, then the solution must be *unique* for each b in $\mathbf{R}^n$.

d) If A and B are invertible $n \times n$ matrices, then $A+B$ is invertible and $(A+B)^{-1} = A^{-1} + B^{-1}$.

e) If A is a 3×4 matrix and B is a 4×2 matrix, then the linear transformation Z defined by $Z(x) = ABx$ has domain $\mathbf{R}^3$ and codomain $\mathbf{R}^2$.

f) Suppose A is an $n \times n$ matrix and every vector in $\mathbf{R}^n$ can be written as a linear combination of the columns of A. Then A must be invertible.

g) If $\det(A) = 1$ and c is a scalar, then $\det(cA) = c\det(A)$.

2. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be rotation *clockwise* by $60°$. Let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation satisfying $U(1,0) = (-2,1)$ and $U(0,1) = (1,0)$.

a) Find the standard matrix for the T and U, and compute the determinant of each matrix.

b) Find the standard matrix for the composition $U \circ T$ using matrix multiplication. Compute the determinant.

c) Find the standard matrix for the composition $T \circ U$ using matrix multiplication. Compute the determinant.

d) Is rotating clockwise by $60°$ and then performing U, the same as first performing U and then rotating clockwise by $60°$?

e) What is the relation between the determinants of these matrices?

3. Let $A = \begin{pmatrix} 7 & 1 & 4 & 1 \\ -1 & 0 & 0 & 6 \\ 9 & 0 & 2 & 3 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

a) Compute $\det(A)$.

b) Compute $\det(A^{-1})$ without doing any more work.

c) Compute $\det((A^T)^5)$ without doing any more work.

d) Find the volume of the parallelepiped formed by the columns of A.