MATH 1553, SPRING 2022
MIDTERM 2

Name	

Circle your lecture below.

Jankowski, lecture A (8:25-9:15 AM) | Jankowski, lecture D (9:30-10:20 AM)

Yu, lecture G (12:30-1:20 PM)

Leykin, lecture I (2:00-2:50 PM) | Leykin, lecture M (3:30-4:20 PM)

Please **read all instructions** carefully before beginning.

- Write your initials at the top of each page.
- The maximum score on this exam is 70 points, and you have 75 minutes to complete this exam. Each problem is worth 10 points.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means “reduced row echelon form."
- Show your work, unless instructed otherwise. A correct answer without appropriate work will receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and all work must be written on the exam itself, with no exceptions.
- This exam is double-sided. You should have more than enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam*. If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

Please read and sign the following statement.

I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 9:15 PM on Wednesday, March 9.

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Problem 1.

For each statement, answer TRUE or FALSE. If the statement is *ever* false, circle FALSE. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

a) Suppose A is a matrix and $Ax = b$ has exactly one solution for some vector b. Then the columns of A are linearly independent.

b) Let V be the subspace of $\mathbf{R}^3$ consisting of all vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ that satisfy $3x - 2y + z = 0$.

Then $\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\}$ is a basis for V.

c) Suppose A is a 20×25 matrix and $\text{rank}(A) = 5$. Then the dimension of the null space of A is 15.

d) Suppose A is a matrix with column vectors v_1, v_2, v_3 that satisfy $v_1 + v_2 = v_3$. Then the matrix transformation $T(x) = Ax$ cannot be one-to-one.

e) If A is a 5×3 matrix and B is a 3×5 matrix, then the matrix transformation given by $T(x) = ABx$ cannot be onto.
TRUE FALSE

Solution.

a) **True**. The solution set for $Ax = b$ is a translation of the solution set to $Ax = 0$, so $Ax = 0$ has exactly one solution, thus the columns of A are linearly independent.

b) **True**. From the fundamentals of chapters 1 and 2, we know that V is a (two-dimensional) plane, and it is quick to verify that the vectors $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ are linearly independent vectors in V, so they form a basis of V by the Basis Theorem.

c) **False**. This problem is #1a from the 2.7+2.9 Supplement with changed numbers. By the Rank Theorem, the rank of A and nullity of A must add to 25, and we were given that the rank of A is 5, so the nullity of A is 20. For other similar problems, see #3 of the 2.7+2.9 Webwork and #2a of the 2.5-3.1 Worksheet.

d) **True**. From the information we are given, the columns of A are linearly dependent since $v_1 + v_2 - v_3 = 0$, so T is not one-to-one.

e) **True**. As we saw in #2c of the 3.4-3.6 Worksheet, we know that since A is 5×3 and B is 3×5, the columns of the 5×5 matrix AB must be linearly dependent since $Bx = 0$ has infinitely many solutions (and thus $ABx = 0$ has infinitely many solutions). Therefore, AB cannot be invertible, so T cannot be onto.

Alternatively, we could note that the column span of AB is contained in the column span of A (also from #2 from the 3.4-3.6 worksheet), which is at most 3-dimensional since A is 5×3. Therefore, the range of T cannot be larger than a 3-dimensional subspace of $\mathbf{R}^5$.

Problem 2.

There is no work required and no partial credit on this page.

a) (3 points) Answer the three questions below.

(I). Let $S = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbf{R}^2 \ : \ ab = 0 \right\}$. Is S a subspace of $\mathbf{R}^2$? YES NO

(II). Let $V = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbf{R}^2 \ : a = b \right\}$. Is V a subspace of $\mathbf{R}^2$? YES NO

(III). Let W be the set of all vectors in $\mathbf{R}^3$ of the form $\begin{pmatrix} a \\ b \\ a-b \end{pmatrix}$, where a and b are real numbers. Is W a subspace of $\mathbf{R}^3$? YES NO

b) (2 points) Suppose $u = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 6 \\ c \end{pmatrix}$.

What values of c will make the set $\{u, v\}$ linearly **dependent**? Clearly circle your answer below.

(i) $c = 0$ only
(ii) $c = 2$ only
(iii) $c = 3$ only
(iv) $c = 9$ only
(v) All c except $c = 2$
(vi) All c except $c = 3$
(vii) All c except $c = 9$

c) (2 points) Suppose $\begin{pmatrix} 1 & -3 & 1 \end{pmatrix}$ is one row in matrix A. Which one of the following vectors **cannot** be in the null space of A? Clearly circle your answer below.

(i) $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ (iii) $\begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$ (iv) $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$

d) (3 points)
Suppose A is a 3×4 matrix and the parametric vector form of the solutions to $Ax = 0$ is

$$x_3 \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}.$$

Which of the following statements must be true? Circle all that apply.

(i) $Ax = b$ is consistent for every b in $\mathbf{R}^3$.

(ii) If b is a vector in the column space of A, then the solution set of $Ax = b$ is a line in $\mathbf{R}^4$.

(iii) The first three columns of A form a basis for Col(A).

Solution.

a) (I) No. This is the example from page 10 of the 2.6 PDF notes, where it is shown that S is not a subspace. This is also an "$\mathbf{R}^2$ version" of #1 from the 2.6 Webwork.

(II) Yes. This is the line $y = x$, i.e. Span$\begin{pmatrix}1\\1\end{pmatrix}$.

(III) Yes. This is inspired by #3 from the 2.6 Webwork. We observe that $W = \text{Span}\left\{\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\-1\end{pmatrix}\right\}$, so W is a subspace of $\mathbf{R}^3$.

b) The answer is $c = 9$ only. In order for the set of nonzero vectors $\{u, v\}$ to be linearly dependent, we need v to be a scalar multiple of u. From the first entries of u and v we see that this requires $v = 3u$, so $c = 3(3) = 9$.

This problem was #2 from the 2.5 Webwork, except that instead of using 3 vectors in $\mathbf{R}^3$ like the Webwork, this problem used 2 vectors in $\mathbf{R}^2$ to make things easier.

c) The answer is (iv). If $Ax = 0$, then the product of any row of A with x is zero. Among the vectors given, the only vector that satisfies $\begin{pmatrix}1 & -3 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \neq 0$ is $x = \begin{pmatrix}2\\0\\2\end{pmatrix}$.

d) This part of the problem tests the Rank Theorem and fundamentals of Col(A).

(i) True. From the information given, the 3×4 matrix A has a 1-dimensional null space, thus A has 3 pivots and therefore A has a pivot in every row.

(ii) True. The solution set to $Ax = b$ is a translation of the solution set to $Ax = 0$, which we were told is Span$\left\{\begin{pmatrix}1\\1\\1\\0\end{pmatrix}\right\}$.

(iii) False. The pivot columns are guaranteed to form a basis for Col A, but since x_3 is a free variable, the third column will **not** have a pivot. One matrix whose null space is the span of $\begin{pmatrix}1\\1\\1\\0\end{pmatrix}$ but whose first three columns are linearly dependent is

$$A = \begin{pmatrix}1 & 0 & -1 & 0\\0 & 1 & -1 & 0\\0 & 0 & 0 & 1\end{pmatrix}.$$

Problem 3.

Parts (a), (b), and (c) are unrelated. You do not need to show your work or justify your answers for (a) and (b), but show your work for part (c).

a) (4 points)

Let $A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \\ 0 & 3 \end{pmatrix}$, and let T be its associated matrix transformation $T(x) = Ax$. Answer the following questions. Clearly circle your answer in parts (i), (ii), and (iii).

(i) What is the domain of T? $\quad \mathbf{R} \quad \mathbf{R}^2 \quad$ a plane in $\mathbf{R}^3 \quad \mathbf{R}^3$

(ii) What is the codomain of T? $\quad \mathbf{R} \quad \mathbf{R}^2 \quad$ a plane in $\mathbf{R}^3 \quad \mathbf{R}^3$

(iii) Are there vectors x and y so that $x \neq y$ but $T(x) = T(y)$? YES NO

(iv) Write one vector v in the codomain of T that is not in the range of T.

b) (2 points) Suppose that $T : \mathbf{R}^3 \to \mathbf{R}^4$ is a one-to-one linear transformation and $T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 0 \end{pmatrix}$. Which one of the following matrices A could possibly be the standard matrix for T? Clearly circle your answer.

(i) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ (ii) $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ (iii) $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \end{pmatrix}$

(iv) $A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ (v) There is no such linear transformation T

c) (4 points) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation that first reflects vectors across the y-axis, then reflects across the line $y = x$. Find the standard matrix A for T.

Solution.

a) This is a modification of #3c from the practice midterm.

(i) The domain of T is $\mathbf{R}^2$.

(ii) The codomain of T is $\mathbf{R}^3$, since every vector in the column space of A is a vector

in $\mathbf{R}^3$.
(iii) No. The columns of A are linearly independent, so T is one-to-one, which means that if $x \neq y$ then $T(x) \neq T(y)$.
(iv) From the form of the columns of A, we see that can get any vector of the form $\begin{pmatrix} a \\ 2b \\ 3b \end{pmatrix}$ where a and b are real numbers. In other words, a vector in $\mathbf{R}^3$ will be in the range of T precisely when its third entry is $3/2$ times its second entry. Here are some vectors in $\mathbf{R}^3$ that are not in the range of T:

$$v = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, \qquad v = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \qquad v = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

Any vector $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ will be a correct answer unless $x_3 = \dfrac{3}{2}x_2$.

b) The correct answer is (i). We could note that (i) satisfies the required properties, or we could use the process of elimination.

Our matrix A must be 4×3 (eliminating (iii)), and A must have a pivot in every column since T is one-to-one (eliminating (ii) and (iv)). The matrix in (i) has a pivot in each column and satisfies $A\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 0 \end{pmatrix}$, so it is our answer.

c) We can either use geometry to compute $T\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $T\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, or use matrix multiplication.

<u>Using geometry</u>

$T\begin{pmatrix} 1 \\ 0 \end{pmatrix}$: reflect across y-axis to get $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$, then reflect across $y = x$ to get $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$.
$T\begin{pmatrix} 0 \\ 1 \end{pmatrix}$: reflect $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ across y-axis and it does not go anywhere (so it stays at $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$), then reflect across $y = x$ to get $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Therefore, the matrix is

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

<u>Using matrix algebra</u>

This is the composition $(T \circ U)(x) = T(U(x))$, where U is reflection across the y-axis and T is reflection across $y = x$. If X is the matrix for T and Y is the matrix for U, then $A = XY$ is the matrix for $T \circ U$:

$$A = XY = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Problem 4.

You do not need to show your work on this problem, and there is no partial credit. Part (a) is 4 points, (b) is 3 points, and (c) is 3 points.

a) For each matrix A below, let T be the matrix transformation $T(x) = Ax$. If T is one-to-one, circle "one-to-one." If T is onto, circle "onto." If T is one-to-one and onto, circle both. If T is neither one-to-one nor onto, do not circle anything.

(I) $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ one-to-one onto

(II) $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ one-to-one onto

(III) $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ one-to-one onto

(IV) $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ one-to-one onto

b) Let A and B be invertible $n \times n$ matrices. Which statements below must be true? Clearly circle all that apply.

(i) $(AB)^{-1} = A^{-1}B^{-1}$.

(ii) $(A+B)(A+B) = A^2 + 2AB + B^2$.

(iii) A and B have the same RREF.

c) Match each of the three matrices below with its corresponding transformation (choosing from (i) through (vii)) by clearly writing that roman numeral in the space provided. We use the usual notation of (x, y) to denote points in $\mathbf{R}^2$.

$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is the standard matrix for ________

$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ is the standard matrix for ________

$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ is the standard matrix for ________

(i) Projection onto the x-axis in $\mathbf{R}^2$.
(ii) Reflection across the y-axis in $\mathbf{R}^2$.
(iii) Reflection across the line $y = x$ in $\mathbf{R}^2$.
(iv) Reflection across the line $y = -x$ in $\mathbf{R}^2$.
(v) Rotation counterclockwise by $\pi/4$ radians in $\mathbf{R}^2$.
(vi) Rotation clockwise by $\pi/4$ radians in $\mathbf{R}^2$.
(vii) Projection onto the y-axis in $\mathbf{R}^2$.

Solution.

a) This is a fundamental problem of counting pivots.
(i) **neither**, since A is 2×2 with 0 pivots.
(ii) **neither**, since A is 2×2 with 1 pivot.
(iii) **onto** (but not one-to-one), since A is 2×3 with 2 pivots.
(iv) **neither**. Quick row-reduction gives $\begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{pmatrix}$, so the 3×3 matrix has only 2 pivots.

b) Parts (i) and (ii) are from #4 of the 3.5+3.6 Webwork. Part (iii) is an application of the Invertible Matrix Theorem or just a consequence of pivot-counting.
(i) **False**. $(AB)^{-1} = B^{-1}A^{-1}$.
(ii) **False**. $(A+B)(A+B) = A^2 + AB + BA + B^2$, but since matrices generally do not commute in multiplication, we cannot guarantee that $AB + BA = 2AB$.
(iii) **True**. Invertible $n \times n$ matrices have a pivot in every row and every column, so their RREF will be the identity matrix.

c) $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is the standard matrix for (vi), rotation clockwise by $\pi/4$ radians in $\mathbf{R}^2$
$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ is the standard matrix for (iv), reflection across the line $y = -x$ in $\mathbf{R}^2$.
$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ is the standard matrix for (vii), projection onto the y-axis in $\mathbf{R}^2$

Problem 5.

Show your work on parts (b) and (c) of this problem.
Consider the matrix A and its reduced row echelon form given below.

$$A = \begin{pmatrix} 1 & -2 & 0 & 1 \\ 1 & -2 & 2 & 3 \\ -1 & 2 & 3 & 2 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

a) (2 points) Write a basis for Col A. There is no work required on this part.

b) (4 points) Find a basis for Nul A.

c) (2 points) Is $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ in the column space of A? Briefly justify your answer.

d) (2 points) Write one nonzero vector in the null space of A. There is no work required and no partial credit for this part.

Solution.

By pure coincidence, the matrix's RREF was almost identical to that of the matrix in #2 in the 3.1 Supplement, which asked similar questions.

a) The pivot columns of A will form a basis for Col(A): $\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} \right\}$. However, in this problem, any two columns of A will form a basis for Col(A) unless we choose $\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ -2 \\ 2 \end{pmatrix} \right\}$.

b) From the RREF of A, for the solution set for $Ax = 0$ we see x_2 and x_4 are free and

$$x_1 - 2x_2 + x_4 = 0, \qquad x_3 + x_4 = 0.$$

Therefore, $x_1 = 2x_2 - x_4$ and $x_3 = -x_4$, and

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2x_2 - x_4 \\ x_2 \\ -x_4 \\ x_4 \end{pmatrix} = x_2 \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix}. \qquad \text{Basis for Nul}(A): \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\}.$$

c) Yes. In fact, $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ is the fourth column of A.

d) Any **nonzero** linear combination of $\begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$ is correct.

Problem 6.

Show your work except on parts (c) and (d) of this problem.

Let $T : \mathbf{R}^2 \to \mathbf{R}^3$ be the transformation given by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 0 \\ x-y \end{pmatrix}$, and let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation of rotation counterclockwise by 90 degrees.

a) (2 points) Write the standard matrix A for T.

b) (2 points) Write the standard matrix B for U.

c) (1 point) Is T one-to-one? YES NO

d) (1 point) Circle the composition that makes sense: $T \circ U$ $U \circ T$.

e) (4 points) Write the standard matrix for the composition you chose above.

Solution.

This problem is a slight modification of #6 from the practice exam.

a) $A = \left(T\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad T \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 1 & -1 \end{pmatrix}$.

b) $B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

c) T is one-to-one.

d) $T \circ U$ makes sense.

e) $AB = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 0 & 0 \\ -1 & -1 \end{pmatrix}$.

Problem 7.

Free response. Show your work!

a) (2 points) Find A^{-1} if $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$.

b) (4 points) Write a matrix A so that the column space of A is the span of $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and the null space of A is the line $y = -x$ in $\mathbf{R}^2$.

c) (4 points) Let $A = \begin{pmatrix} 1 & 0 & -2 \\ 1 & 1 & 0 \\ 0 & 2 & k \end{pmatrix}$. Find all values of k so that A is invertible.

Solution.

a) $A^{-1} = \dfrac{1}{2(4)-3(1)} \begin{pmatrix} 4 & -1 \\ -3 & 2 \end{pmatrix} = \dfrac{1}{5} \begin{pmatrix} 4 & -1 \\ -3 & 2 \end{pmatrix}$.

b) A must be 3×2 since its column space lives in $\mathbf{R}^3$ and its null space lives in $\mathbf{R}^2$. Every column must be a scalar multiple of $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, and we want the RREF to be $\begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$.

One example is $A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 1 \end{pmatrix}$.

c) Our answer is all real numbers k satisfying $k \neq 4$.

$$\left(A \mid 0\right) = \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & k & 0 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & k & 0 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & k-4 & 0 \end{array}\right).$$

This matrix will have 3 pivots unless the second and third rows above are scalar multiples of each other. This means $k = 2(2)$, so $k = 4$. Therefore, the matrix is invertible **unless** $k = 4$.

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.