Math 1553 Worksheet §5.2 - §5.4

1. Suppose A is an $n \times n$ matrix satisfying $A^2 = 0$. Find all eigenvalues of A. Justify your answer.

2. Answer yes, no, or maybe. Justify your answers. In each case, A is a matrix whose entries are real numbers.

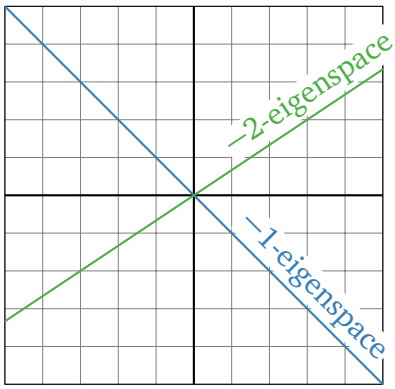
a) Suppose $A = \begin{pmatrix} 3 & 0 & 0 \\ 5 & 1 & 0 \\ -10 & 4 & 7 \end{pmatrix}$. Then the characteristic polynomial of A is

$$\det(A - \lambda I) = (3-\lambda)(1-\lambda)(7-\lambda).$$

b) If A is a 3×3 matrix with characteristic polynomial $-\lambda(\lambda-5)^2$, then the 5-eigenspace is 2-dimensional.

c) If A is an invertible 2×2 matrix, then A is diagonalizable.

3. The eigenspaces of some 2×2 matrix A are drawn below. Write an invertible matrix C and a diagonal matrix D so that $A = CDC^{-1}$. Can you find another pair of C and D so that $A = CDC^{-1}$?



4. Suppose A is a 2×2 matrix satisfying

$$A\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}2\\-2\end{pmatrix}, \qquad A\begin{pmatrix}-2\\3\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}.$$

a) Diagonalize A by finding 2×2 matrices C and D (with D diagonal) so that $A = CDC^{-1}$.

b) Find A^{17}.