MATH 1553, FALL 2023
SAMPLE MIDTERM 1A: COVERS THROUGH SECTION 2.4

Name	

Please **read all instructions** carefully before beginning.

- Write your name at the top of each page (not just the cover page!).
- The maximum score on this exam is 70 points, and you have 75 minutes to complete this exam. Each problem is worth 10 points.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means "reduced row echelon form."
- Show your work, unless instructed otherwise. A correct answer without appropriate work will receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and all work must be written on the exam itself, with no exceptions.
- This exam is double-sided. You should have more than enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page*. If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

This is a practice exam. It is meant to be similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. We recommend completing the practice exam in 75 minutes, without notes or distractions.

This page was intentionally left blank.

Problem 1.

TRUE or FALSE. If the statement is *ever* false, circle FALSE. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

a) Suppose we are given a consistent system of 1 linear equation in 3 variables and the corresponding augmented matrix has 1 pivot in its reduced row echelon form. Then the set of solutions to the equation must be a line.

TRUE FALSE

b) The following vector equation is consistent for every b in $\mathbf{R}^3$:

$$x_1 \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = b.$$

TRUE FALSE

c) Suppose u, v, and w are vectors in $\mathbf{R}^3$. Then $\text{Span}\{u, v, w\}$ is either a plane in $\mathbf{R}^3$ or all of $\mathbf{R}^3$.

TRUE FALSE

d) If A is an $m \times n$ matrix with more columns than rows, then $Ax = b$ must be inconsistent for some b in $\mathbf{R}^m$.

TRUE FALSE

e) Suppose A is a 2×2 matrix and b is a vector in $\mathbf{R}^2$. If the solution set to $Ax = b$ is the span of $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$, then $b = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

TRUE FALSE

Problem 2.

Multiple choice and short answer. You do not need to show work or justify your answers. Parts (a), (b), and (c) are unrelated.

a) (3 points) Which of the following equations are linear equations in the variables x, y, and z? Clearly circle LINEAR or NOT LINEAR in each case.

(i) $x - yz = 1$ LINEAR NOT LINEAR

(ii) $9x - 5y + 17z = 7$ LINEAR NOT LINEAR

(iii) $2x - y + \sqrt{z} = 0$ LINEAR NOT LINEAR

b) (4 points) Which of the following matrices are in reduced row echelon form (RREF)? Clearly circle all that apply.

(i) $\left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$

(ii) $\left(\begin{array}{ccc|c} 0 & 1 & 2 & 3 \end{array}\right)$

(iii) $\left(\begin{array}{cccc|c} 1 & 0 & 1 & 3 & 0 \\ 0 & 1 & -2 & 3 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right)$

(iv) $\left(\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)$

c) (3 points) Write a *vector equation* that corresponds to a system of 2 linear equations in 2 variables (x_1 and x_2) with **infinitely many solutions**.

Problem 3.

Short answer and multiple choice. Briefly show your work in part (a). Parts (a), (b), (c), and (d) are unrelated.

a) (2 points) Compute the product $\begin{pmatrix} 1 & -1 & -5 \\ 3 & 4 & -7 \end{pmatrix}\begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$.

b) (2 pts) Write a set of three *different* vectors u, v, and w whose span is a line in $\mathbf{R}^3$.

c) (4 points) Suppose A is a matrix and b is a vector in $\mathbf{R}^4$ so that the set of solutions to $Ax = b$ has the parametric vector form given below, where x_3 is a free variable:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}.$$

(i) Which of the following are solutions to the equation $Ax = b$? Circle all that apply.

(I) $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ (II) $\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

(ii) How many rows does A have? Circle your answer below.

1 2 3 4 not enough information

(iii) How many columns does A have? Circle your answer below.

1 2 3 4 not enough information

d) (2 points) Let v_1, v_2, w be vectors in $\mathbf{R}^3$, and suppose that the matrix whose three columns are v_1, v_2, and w has three pivots. Which of the following statements must be true? Clearly circle all that apply.

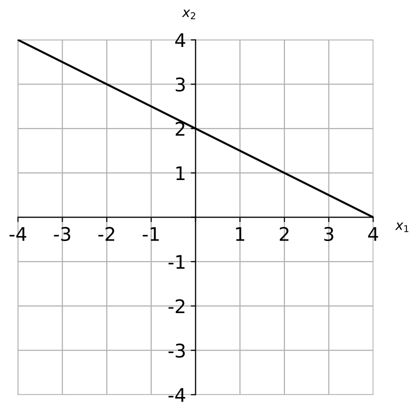
(i) Every vector in $\mathbf{R}^3$ is a linear combination of v_1, v_2, and w.

(ii) w is not a linear combination of v_1 and v_2.

Problem 4.

Short answer. You do not need to show your work on this page. Parts (a), (b), and (c) are unrelated.

a) (3 points) For some matrix A and some vector b, the diagonal line below is the solution set for $Ax = b$. On the same graph, draw the solution set for the homogeneous system $Ax = 0$.



b) (4 points) Suppose we are given a system of 3 linear equations in 3 variables. Which of the following statements are true? Clearly circle all that apply.

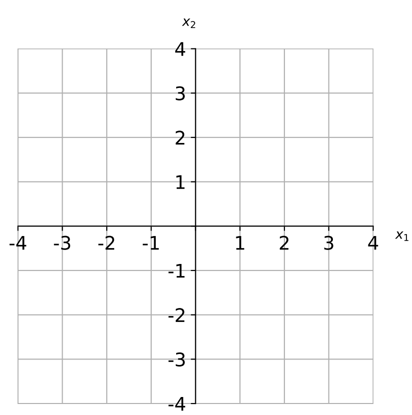
(i) If the system is consistent, then it must have exactly one solution.

(ii) The system must be consistent if its corresponding augmented matrix has 3 pivots.

(iii) The system must be consistent if the RREF of the corresponding augmented matrix has *bottom row* equal to $\left(\begin{array}{ccc|c} 0 & 0 & 1 & -2 \end{array}\right)$.

(iv) One solution to the system must be the trivial solution $x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

c) (3 points) Let $A = \begin{pmatrix} 1 & 2 \\ -2 & -4 \end{pmatrix}$. Graph the span of the columns of A below. Make sure your graph is clear and complete!



The rest of the exam is free response. Unless told otherwise, show your work!

Problem 5.

The two parts of this problem are unrelated.

a) (5 pts) Consider the linear system of equations given by

$$3x + 10y = 8$$
$$9x - hy = k$$

Find all values of h and k (if there are any) so that the system has **no solution**.

b) (5 points) Find all real numbers c so that $\begin{pmatrix} -2 \\ 3 \\ c \end{pmatrix}$ is in $\operatorname{Span}\left\{ \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \\ 7 \end{pmatrix} \right\}$.

Problem 6.

Free response. Show your work in parts (a) and (b). You do not need to show your work on part (c).

Consider the following linear system of equations in the variables x_1, x_2, x_3, x_4.

$$x_1 + 4x_2 - x_3 - 4x_4 = 2$$
$$2x_1 + 8x_2 - x_3 - 7x_4 = 4$$
$$5x_1 + 20x_2 - x_3 - 16x_4 = 10.$$

a) (4 points) Write the augmented matrix corresponding to this system, and put the augmented matrix into reduced row echelon form (RREF).

b) (4 points) This system is consistent. Write the set of solutions to the system of equations in parametric vector form.

c) (2 points) Write one vector that is *not* the zero vector but is a solution to the corresponding **homogeneous** system of equations given below.

$$x_1 + 4x_2 - x_3 - 4x_4 = 0$$
$$2x_1 + 8x_2 - x_3 - 7x_4 = 0$$
$$5x_1 + 20x_2 - x_3 - 16x_4 = 0.$$

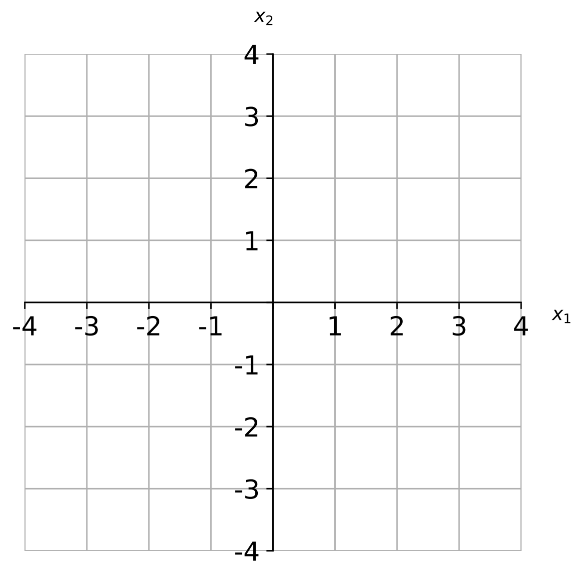
Problem 7.

Free response. The two parts of this problem are unrelated. In this problem, we use the usual convention of $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ to represent vectors in $\mathbf{R}^2$.

a) (6 points) Consider the matrix equation $Ax = b$, where

$$A = \begin{pmatrix} -1 & 3 \\ 2 & -6 \end{pmatrix}, \qquad b = \begin{pmatrix} 2 \\ -4 \end{pmatrix}.$$

Find the solution set to $Ax = b$ and draw it on the graph below.



b) (4 points) Write an augmented matrix in RREF that corresponds to a system of linear equations in the variables x_1 and x_2 whose solution set has parametric form

$$x_1 = 1 - 4x_2, \qquad x_2 = x_2 \quad (x_2 \text{ free}).$$

Briefly justify why your matrix satisfies these conditions.

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.