Math 1553 Reading Day Fall 2023

This is a preview of the published version of the quiz

Started: Nov 4 at 11:03am

Quiz Instructions

Question 1

1 pts

If $\{u, v, w\}$ is a set of linearly dependent vectors, then w must be a linear combination of u and v.

- True
- False

Question 2

1 pts

Find the value of k that makes the following vectors linearly dependent:

$$\begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ -3 \\ k \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$$

Question 3

1 pts

If $\{u, v\}$ is a basis for a subspace W, then $\{u - v, u + v\}$ is also a basis for W.

- True
- False

Question 4 1 pts

Which of the following are subspaces of $\mathbb{R}^4$?

(1) The set $W = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbb{R}^4 \;:\; 2x - y - z = 0 \right\}$.

(2) The set of solutions to the equation $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 3 & 0 & -1 \end{pmatrix} x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

- both are subspaces
- neither is a subspace
- (2) is a subspace but (1) is not a subspace
- (1) is a subspace but (2) is not a subspace

Question 5 1 pts

Let W be the set of vectors $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ in $\mathbb{R}^3$ with $abc = 0$. Then W is closed under addition, meaning that if v and w are in W, then $v + w$ is in W.

- True
- False

Question 6

1 pts

Match the transformations given below with their corresponding 2×2 matrix.

A. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

B. $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

C. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

D. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

E. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Counter-clockwise rotation by 90 degrees	[Choose]
Reflection about the line y=x	[Choose]
Clockwise rotation by 90 degrees	[Choose]
Reflection across the x-axis	[Choose]
Reflection across the y-axis	[Choose]

Question 7

1 pts

Find the value of k so that the matrix transformation for the following matrix is not onto.

$$\begin{pmatrix} 1 & 3 & 9 \\ 2 & 6 & k \end{pmatrix}$$

Question 8

1 pts

Find the **nonzero** value of k that makes the following matrix not invertible.

$$\begin{pmatrix} 1 & -1 & 0 \\ k & k^2 & 0 \\ -1 & 1 & 5 \end{pmatrix}$$

Enter an integer as your answer. Note that 0 is not the correct answer, since the question asks for a nonzero value of k.

Question 9

1 pts

Match the following definitions with the corresponding term describing a linear transformation $T : \mathbb{R}^m \to \mathbb{R}^n$.

Each definition should be used exactly once.

A. For each y in $\mathbb{R}^n$ there is at most one x in $\mathbb{R}^m$ so that $T(x) = y$.

B. For each y in $\mathbb{R}^n$ there is at least one x in $\mathbb{R}^m$ so that $T(x) = y$.

C. For each y in $\mathbb{R}^n$ there is exactly one x in $\mathbb{R}^m$ so that $T(x) = y$.

D. For each x in $\mathbb{R}^m$ there is exactly one y in $\mathbb{R}^n$ so that $T(x) = y$.

T is a transformation [Choose]

T is one-to-one

[Choose]

T is onto [Choose]

T is one-to-one and onto [Choose]

Question 10

1 pts

Suppose A is a 4×6 matrix. Then the dimension of the null space of A is at most 2.

- True
- False

Question 11

1 pts

Complete the entries of the matrix A so that $\mathrm{Col}(A) = \mathrm{Span}\left\{\begin{pmatrix}1\\2\end{pmatrix}\right\}$ and $\mathrm{Nul}(A) = \mathrm{Span}\left\{\begin{pmatrix}1\\1\end{pmatrix}\right\}$.

$A = \begin{pmatrix} r & 1 \\ s & 2 \end{pmatrix}$, where r = [ ] and s = [ ]

Question 12

1 pts

Suppose $T : \mathbb{R}^7 \to \mathbb{R}^9$ is a linear transformation with standard matrix A, and suppose that the range of T has a basis consisting of 3 vectors. What is the dimension of the null space of A?

Question 13

1 pts

Define a transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ by $T(x, y, z) = (0,\ x - y,\ y - x,\ z)$.

Which *one* of the following statements is true?

- ○ T is onto but not one-to-one.
- ○ T is one-to-one but not onto.
- ○ T is one-to-one and onto.
- ○ T is neither one-to-one nor onto.

Question 14

1 pts

Suppose that A is a 7×5 matrix, and the null space of A is a line. Say that T is the matrix transformation $T(v) = Av$. Which of the following statements must be true about the range of T?

- ○ It is a 4-dimensional subspace of $\mathbb{R}^5$
- ○ It is a 6-dimensional subspace of $\mathbb{R}^7$
- ○ It is a 4-dimensional subspace of $\mathbb{R}^7$
- ○ It is a 6-dimensional subspace of $\mathbb{R}^5$

Question 15

1 pts

Say that $S : \mathbb{R}^2 \to \mathbb{R}^3$ and $T : \mathbb{R}^3 \to \mathbb{R}^4$ are linear transformations. Which of the following must be true about $T \circ S$?

- ◯ It is one-to-one
- ◯ It is not one-to-one
- ◯ It is onto
- ◯ The composition is not defined
- ◯ It is not onto

Question 16

1 pts

Suppose that A is an invertible $n \times n$ matrix. Then $A + A$ must be invertible.

- ◯ True
- ◯ False

Question 17

1 pts

Suppose A is a 3×3 matrix and the equation $Ax = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$ has exactly one solution.

Then A must be invertible.

- ◯ True
- ◯ False

Question 18

1 pts

Suppose that A and B are $n \times n$ matrices and AB is not invertible.

Which *one* of of the following statements must be true?

- None of these
- B is not invertible
- At least one of the matrices A or B is not invertible
- A is not invertible

Question 19

1 pts

Suppose A and B are 3×3 matrices, with $\det(A) = 3$ and $\det(B) = -6$.

Find $\det(2A^{-1}B)$.

Question 20

1 pts

Let A be the 3×3 matrix satisfying $Ae_1 = e_3$, $Ae_2 = e_2$, and $Ae_3 = 2e_1$ (recall that we use e_1, e_2, and e_3 to denote the standard basis vectors for $\mathbb{R}^3$).

Find $\det(A)$.

Question 21

1 pts

Suppose A is a square matrix and $\lambda = -1$ is an eigenvalue of A.

Which one of the following statements must be true?

- ○ $\text{Nul}(A + I) = \{0\}$
- ○ The columns of $A + I$ are linearly independent.
- ○ A is invertible.
- ○ For some nonzero x, the vectors Ax and x are linearly dependent.
- ○ The equation \(Ax = x \\)has only the trivial solution.

Question 22

1 pts

Suppose A is a 4 x 4 matrix with characteristic polynomial $-(1-\lambda)^2(5-\lambda)\lambda$.

What is the rank of A?

Question 23

1 pts

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that reflects across the line $x_2 = 2x_1$.

Find the value of k so that $A\begin{pmatrix} 2 \\ k \end{pmatrix} = \begin{pmatrix} 2 \\ k \end{pmatrix}$.

Question 24

1 pts

Find the value of k such that the matrix $\begin{pmatrix} 1 & k \\ 1 & 3 \end{pmatrix}$ has one real eigenvalue of algebraic multiplicity 2. *Enter an integer value below.*

Question 25

1 pts

Suppose that A is a 5×5 matrix with characteristic polynomial $(1-\lambda)^3(2-\lambda)(3-\lambda)$ and also that A is diagonalizable. What is the dimension of the 1-eigenspace of A?

Question 26

1 pts

Find the value of t such that 3 is an eigenvalue of $\begin{pmatrix} 1 & t & 3 \\ 1 & 1 & 1 \\ 3 & 0 & 3 \end{pmatrix}$. *Enter an integer answer below.*

Question 27

1 pts

Say that A is a 2×2 matrix with characteristic polynomial $(1-\lambda)(2-\lambda)$. What is the characteristic polynomial of A^2?

- ○ $(1-\lambda)^2(2-\lambda)^2$
- ○ $(1-\lambda^2)(2-\lambda^2)$
- ○ $(1-\lambda^2)(4-\lambda^2)$
- ○ $(1-\lambda)(2-\lambda)$
- ○ $(1-\lambda)(4-\lambda)$

Question 28

1 pts

Suppose that a vector x is an eigenvector of A with eigenvalue 3 and that x is also an eigenvector of B with eigenvalue 4. Which of the following is true about the matrix $2A-B$ and x:

- ○ x is an eigenvector of $2A-B$ with eigenvalue 3
- ○ x is an eigenvector of $2A-B$ with eigenvalue 2
- ○ x is an eigenvector of $2A-B$ with eigenvalue 1
- ○ x is an eigenvector of $2A-B$ with eigenvalue 4
- ○ None of these

Question 29

1 pts

Suppose that A is a 4×4 matrix with eigenvalues 0, 1, and 2, where the eigenvalue 1 has algebraic multiplicity two.

Which of the following must be true?

(1) A is not diagonalizable

(2) A is not invertible

- Both (1) and (2) must be true
- Neither statement is necessarily true
- (2) must be true but (1) might not be true
- (1) must be true but (2) might not be true

Question 30

1 pts

Suppose$A$ is a 5×5 matrix whose entries are real numbers. Then A must have at least one real eigenvalue.

- True
- False

Question 31

1 pts

Suppose A is a positive stochastic matrix and $A \begin{pmatrix} 3/5 \\ 2/5 \end{pmatrix} = \begin{pmatrix} 3/5 \\ 2/5 \end{pmatrix}$. Let $v = \begin{pmatrix} 5 \\ 95 \end{pmatrix}$.

As n gets very large, $A^n v$ approaches the vector $\begin{pmatrix} r \\ s \end{pmatrix}$, where:

r = _____ and s = _____ .

Question 32

1 pts

Suppose that A is a 4×4 matrix of rank 2. Which one of the following statements must be true?

- ○ A cannot have four distinct eigenvalues
- ○ A is not diagonalizable
- ○ none of these
- ○ A is diagonalizable
- ○ A must have four distinct eigenvalues

Question 33

1 pts

Suppose A is a 2×2 matrix whose entries are real numbers, and suppose A has eigenvalue $1+i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1+i \end{pmatrix}$.

Which of the following must be true?

- ○ A must have eigenvalue $1-i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1+i \end{pmatrix}$
- ○ A must have eigenvalue $1-i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1-i \end{pmatrix}$
- ○ None of these
- ○ A must have eigenvalue $1+i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1-i \end{pmatrix}$

Question 34

1 pts

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that rotates the plane clockwise by 45 degrees, and let A be the standard matrix for T.

Which *one* of the following statements is true?

- ○ A has two distinct real eigenvalues
- ○ A has one complex eigenvalue with algebraic multiplicity two
- ○ A has one real eigenvalue with algebraic multiplicity two
- ○ A has two distinct complex eigenvalues.

Question 35

1 pts

Suppose u and v are orthogonal unit vectors (to say that a vector is a unit vector means that it has length 1). Find the dot product

$$(3u - 8v) \cdot 4u.$$

Question 36

1 pts

Find the value of k that makes the following pair of vectors orthogonal.

$$\begin{pmatrix} 2 \\ k \\ 1 \end{pmatrix} \; and \; \begin{pmatrix} k \\ 1 \\ -6 \end{pmatrix}$$

Your answer should be an integer.

Question 37

1 pts

If W is a subspace of $\mathbb{R}^{100}$ and v is a vector in $W^{\perp}$ then the orthogonal projection of v to W must be the 0 vector.

- True
- False

Question 38

1 pts

Suppose W is a subspace of $\mathbb{R}^n$. If x is a vector and x_W is the orthogonal projection of x onto W, then $x \cdot x_W$ must be 0.

- True
- False

Question 39

1 pts

Suppose that A is a 3×3 invertible matrix. What is the dot product between the second row of A and third column of A^{-1} equal to?

- 1
- Not Enough Information is Given
- 2
- -2
- -1
- 0

Question 40

1 pts

Find the orthogonal projection of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ onto $\mathrm{Span}\left\{\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right\}$.

The orthogonal projection is $\begin{pmatrix} a \\ b \end{pmatrix}$, where: a = [ ] and b = [ ].

Enter integers or fractions as your entries.

Question 41

1 pts

Compute the orthogonal projection of the vector $\begin{pmatrix} 6 \\ 5 \\ 4 \end{pmatrix}$ to the plane spanned by the vectors $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$. What is the first coordinate of the projection? *Your answer should be an integer.*

[ ]

Question 42

1 pts

Suppose B is the standard matrix for the transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ of orthogonal projection onto the subspace $W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ in } \mathbb{R}^3 \;\middle|\; x + y + 2z = 0 \right\}$.

What is the dimension of the 1-eigenspace of B?

[ ]

Question 43

1 pts

Let W be the subspace of $\mathbb{R}^4$ given by all vectors $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$ such that $x - y + z + w = 0$. Find dimension of the orthogonal complement $W^\perp$.

Question 44

1 pts

If b is in the column space of the matrix A then every solution to $Ax = b$ is a least squares solution.

- True
- False

Question 45

1 pts

If A is an $m \times n$ matrix, b is in $\mathbb{R}^m$, and $\hat{x}$ is a least squares solution to $Ax = b$, then $\hat{x}$ is the point in $\mathrm{Col}(A)$ that is closest to b.

- True
- False

Question 46

1 pts

Find the least squares solution $\hat{x}$ to the linear system

$$\begin{pmatrix} 6 \\ -2 \\ -2 \end{pmatrix} x = \begin{pmatrix} 14 \\ -2 \\ 0 \end{pmatrix}.$$

If your answer is an integer, enter an integer.

If your answer is not an integer, enter a fraction.

Question 47

1 pts

Find the best fit line y= ______ x+ ______ for the data points $(-7, -22)$, $(0, -2)$, and $(7, 6)$ using the method of least squares. *Your answers should both be integers.*

Question 48

1 pts

Let $A = \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} -3 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix}^{-1}$.

Find r and s so that $A^3 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} r \\ s \end{pmatrix}$.

r = ______

s = ______

Question 49

1 pts

If A is a diagonalizable 6×6 matrix, then A has 6 distinct eigenvalues.

- True
- False

Question 50

1 pts

Find the eigenvalues of the matrix $A = \begin{pmatrix} 1 & 4 \\ 4 & 7 \end{pmatrix}$ and write them in increasing order.

The smaller eigenvalue is λ_1 = ______ .

The larger eigenvalue is λ_2 = ______ .

Not saved Submit Quiz