Math 1553 Worksheet §3.4

1. True or false. Answer true if the statement is *always* true. Otherwise, answer false. If your answer is false, either give an example that shows it is false or (in the case of an incorrect formula) state the correct formula.

a) If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in $\mathbf{R}^n$, then the solution is *unique* for each b in $\mathbf{R}^n$.

b) If A is a 3×4 matrix and B is a 4×2 matrix, then the linear transformation Z defined by $Z(x) = ABx$ has domain $\mathbf{R}^3$ and codomain $\mathbf{R}^2$.

c) Suppose A and B are matrices so that the product AB is defined, and suppose that the transformations $T(v) = Av$ and $U(x) = Bx$ are one-to-one. Then the transformation $T \circ U$ must also be one-to-one.

2. A is $m \times n$ matrix, B is $n \times m$ matrix. Select all correct answers from the box. It is possible to have more than one correct answer.

a) Suppose x is in $\mathbf{R}^m$. Then ABx *must be* in:

Col(A), Nul(A), Col(B), Nul(B)

b) Suppose x in $\mathbf{R}^n$. Then BAx *must be* in:

Col(A), Nul(A), Col(B), Nul(B)

c) If $m > n$, then columns of AB could be linearly $\boxed{independent, \quad dependent}$

d) If $m > n$, then columns of BA could be linearly $\boxed{independent, \quad dependent}$

e) If $m > n$ and $Ax = 0$ has nontrivial solutions, then columns of BA could be linearly $\boxed{independent, \quad dependent}$

3. Consider the following linear transformations:

$T : \mathbf{R}^3 \longrightarrow \mathbf{R}^2$ T projects onto the xy-plane, forgetting the z-coordinate

$U : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ U rotates clockwise by $90°$

$V : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ V scales the x-direction by a factor of 2.

Let A, B, C be the matrices for T, U, V, respectively.

a) Write A, B, and C.

b) Compute the matrix for $V \circ U \circ T$.

c) Describe geometrically the transformation U^{-1} that would undo "U" in the sense that $(U^{-1} \circ U)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$. Now, do the same for V. (we will study these in sections 3.5 and 3.6, and they are called "inverses")