Math 1553 Worksheet §5.2 - §5.4

1. Answer yes, no, or maybe. Justify your answers. In each case, A is a matrix whose entries are real numbers.

   **a)** Suppose $A = \begin{pmatrix} 3 & 0 & 0 \\ 5 & 1 & 0 \\ -10 & 4 & 7 \end{pmatrix}$. Then the characteristic polynomial of A is
   $$\det(A - \lambda I) = (3 - \lambda)(1 - \lambda)(7 - \lambda).$$

   b) If A is a 3×3 matrix with characteristic polynomial $-\lambda(\lambda - 5)^2$, then the 5-eigenspace is 2-dimensional.

   c) If A is an invertible 2×2 matrix, then A is diagonalizable.

2. The eigenspaces of some 2×2 matrix A are drawn below. Write an invertible matrix C and a diagonal matrix D so that $A = CDC^{-1}$. Can you find another pair of C and D so that $A = CDC^{-1}$?
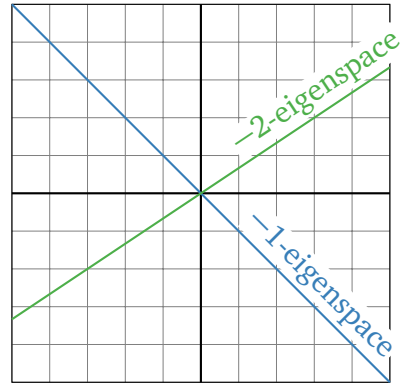
3. Suppose A is a 2×2 matrix satisfying

$$A\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \qquad A\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

a) Diagonalize A by finding 2×2 matrices C and D (with D diagonal) so that $A = CDC^{-1}$.

b) Find A^{17}.