

Math 1553 Worksheet §5.4-5.6

1. True or false. Justify your answer.

A 3×3 matrix A can have a non-real complex eigenvalue with multiplicity 2.

2. Let $A = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}^{-1}$, and let $x = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. What happens to $A^n x$ as n gets very large?

3. Let $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$. Find all eigenvalues of A . For each eigenvalue, find an associated eigenvector.

4. A video game offers participants the chance to play as one of two characters: Archer or Barbarian. The game has 100 million players.

In 2023:

Archer is played by 60 million players.

Barbarian is played by 40 million players.

One year later, in 2024:

- 60% of the people who started with the Archer still play with the Archer, while 40% have switched to Barbarian.
 - 70% of the customers who started with the Barbarian still play with the Barbarian, while 30% have switched to Archer.
- a) Write down the stochastic matrix A which represents the change in each character's popularity from 2023 to 2024, and use it to find the number of people who played with each character in 2024.

- b) Suppose the trend continues each year. In the distant future, who will be the most popular character? What will be the player distribution?