Math 1553 Worksheet §1.2, §1.3

1. Circle the 'operations' that are legal to use in row reduction, in other words, the operations that will not change the solution set of an arbitrary linear system.
   (1) $R_2 = R_3 - 3R_2$
   (2) $R_3 = 3R_3$
   (3) $R_1 \leftrightarrow R_2$
   (4) $R_1 = R_2 - R_3$
   (5) $R_2 = R_2 - R_1^2$
   (6) $R_3 = R_3 - \sqrt{R_1}$

   Additional Question: These are row operations only. You cannot perform column operations, as that will change the solution set of the linear system. For example, try doubling any column in $\left(1 \mid 1 \right)$. What happens to the solution set?

2. a) Which of the following matrices are in row echelon form (REF)? Which are in reduced row echelon form (RREF)?

   b) For the matrices that are in REF or RREF, which entries are the pivots? What are the pivot columns?

   $$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right) \qquad \left(\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{array} \right) \qquad \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

   c) Why is RREF useful, i.e. what information does it reveal about the linear system?

   d) How many nonzero entries are there in a pivot column of a matrix that is in RREF?

3. Each matrix below is in RREF. In each case, determine whether the corresponding system of linear equations is consistent, and if so, how many solutions does it have?

(a) $\begin{pmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 1 \end{pmatrix}$,
(b) $\begin{pmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 0 & | & 1 \end{pmatrix}$,
(c) $\begin{pmatrix} 1 & 0 & 0 & 0 & | & 0 \\ 0 & 1 & 2 & 0 & | & 0 \\ 0 & 0 & 0 & 1 & | & 5 \end{pmatrix}$,
(d) $\begin{pmatrix} 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$

4. Find the parametric form for the solution set of the following system of linear equations in x_1, x_2, and x_3 by putting an augmented matrix into reduced row echelon form. State which variables (if any) are free variables. Describe the solution set geometrically.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ -4x_1 - 9x_2 + 2x_3 &= -1 \\ -3x_2 - 6x_3 &= -3. \end{aligned}$$