Math 1553 Worksheet §3.2, 3.3

1. Which of the following statements are true? Justify your answer.

a) Let A be a 3×3 matrix, such that $Ax = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$ has a unique solution. Then, $Ax = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$ also has a unique solution.

b) The transformation $T(x, y, z) = (y, 4x + z, x)$ is an onto linear transformation.

c) Let A be a 3×4 matrix. Then, the transformation whose standard matrix is A cannot be onto.

2. Which of the following transformations T are onto? Which are one-to-one? If the transformation is not onto, find a vector not in the range. If the transformation is not one-to-one, find two vectors with the same image.

a) Counterclockwise rotation by 32° in $\mathbf{R}^2$.

b) The transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ defined by $T(x, y, z) = (z, x)$.

c) The transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ defined by $T(x, y, z) = (0, x)$.

d) The matrix transformation with standard matrix $A = \begin{pmatrix} 1 & 6 \\ -1 & 2 \\ 2 & -1 \end{pmatrix}$.

e) The matrix transformation with standard matrix $A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.

3. The second little pig has decided to build his house out of sticks. The big bad wolf finds the pig's house and blows it down so that the house is rotated by an angle of 45° in a counterclockwise direction about the z-axis (look downward onto the xy- plane the way we usually picture the plane as $\mathbf{R}^2$), and then projected onto the xy-plane. Find the standard matrix A for the transformation T caused by the wolf.

4. This is extra practice in case the studio finishes the rest of the worksheet early.

On your computer, go to the Interactive Transformation Challenge! Complete the Zoom, Reflect, and Scale challenges. If you complete a challenge in the optimal number of steps, the interactive demo will congratulate you. See if you can complete each of these challenges in the optimal number of steps.