

Study Sheet for Math 3215, Exam 1, Fall 2011

October 3, 2011

1. Know basic properties of sets – complements, unions, intersections, de Morgan’s Laws, the “universal set”. Know all the basic symbols of set theory (e.g. \in, \cap, \cup etc.)
2. Know basic counting principles, such as the “multiplication principle”. Know the difference between permutations and combinations, and how the formulas for each and how they were derived.
3. Know the definitions of “sample space”, “event”, “elementary event”.
4. Know the Kolmogorov axioms of probability: If \mathcal{S} is the sample space, then
 - i) $\mathbb{P}(\mathcal{S}) = 1$;
 - 2) If $E \subseteq \mathcal{S}$, then $\mathbb{P}(E) \geq 0$;
 - 3) If A_1, A_2, \dots is at most a countable number of events, pairwise disjoint, then $P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$.
5. Know how to prove the inclusion-exclusion formula for two events, and know what it says for three or more events. Know how to work problems using it.
6. Know the “union bound” $P(A_1 \cup A_2 \cup \dots) \leq \sum_i P(A_i)$, even if the A_i ’s are not disjoint.
7. Know the definition of independent events. Know the difference between “pairwise independent events” and “mutually independent events”. Know how to prove various consequences of independence, such as: A, B independent implies $P(\overline{A} \cap \overline{B}) = P(\overline{A})P(\overline{B})$.
8. Know the difference between “disjoint events” and “independent events”.
9. Know the definition of conditional probability $\mathbb{P}(A|B)$.

10. Know Bayes's Rule and the "product rule (or chain rule)", and how to apply them to problems. The chain rule says that if A_1, \dots, A_k are events – dependent, independent... doesn't matter – then

$$\mathbb{P}(A_1, \dots, A_k) = \mathbb{P}(A_1)\mathbb{P}(A_2|A_1)\mathbb{P}(A_3|A_1, A_2) \cdots \mathbb{P}(A_k|A_1, \dots, A_{k-1}).$$

11. Know the definition of a random variable. Know the definition of probability density functions (pdf's for short), and know the definition of a cumulative distribution function. Know that $F'(x) = f(x)$, where F is the cdf and f is the pdf.
12. Know what the "support" of a random variable is.
13. Know the following basic random variables, and how to work problems using them:
- a) Bernoulli (flip a single coin)
 - b) Poisson (and know Poisson Processes; know the example I worked in class about the fastfood drive-thru)
 - c) Binomial (flip n contains, p = chance of heads, $q = 1-p$ = chance of tails)
 - d) Geometric (if at first you don't succeed, try, try again)
 - e) Negative Binomial (try until fail/succeed r times)
 - f) Hypergeometric (draw balls from urns without replacement).
14. Know how to compute expectation (mean) and variance of a r.v. Know the formula for variance in terms of the second moment and the mean. Know properties of expectation and variance, such as linearity (for expectation) and that $V(aX) = a^2V(X)$ and $V(X + t) = V(X)$.
15. Know Markov's inequality and Chebychev's inequality.
16. Know how to compute the pdf for a random variable Y that is a function of some other random variable X . The idea is to pass to cdf's by finding $P(Y \leq t)$, and then take a derivative.