

Math 3215 Homework 2, part 2

September 28, 2011

1. You flip a fair coin twice and then a biased coin twice; each flip is assumed to be independent of the others. Suppose that the biased coin has a 40% chance of coming up ‘heads’, and a 60% chance of coming up ‘tails’. Let X be the random variable equal to the total number of heads you flip on all four flips combined.
 - a. Determine the support of X .
 - b. Determine the mass function for X .
2. You have two urns, both with 10 balls (20 balls in total, in other words). Whenever you draw a ball from the first urn, you always replace it; but when you draw a ball from the second urn, you do *not* replace it. Suppose you draw 5 balls from the two urns (no ball is preferred over any other). What is the probability that exactly 3 of the balls you drew came from urn 1 (while the remaining 2 came from urn 2)?
3. Suppose you have two ants that start at the origin $(0, 0)$ at time $t = 0$, and then each either hops to the right one unit or up one unit by time $t = 1$; then, they either hop up or right again by time $t = 2$; and so on. Assuming that the hops of each ant is independent of the hops of the other ant, and even to its own combination of previous hops. Determine the probability that at time $t = 10$ both ants will be at position $(5, 5)$.

By “hopping up or to the right”, I mean that the position of an ant may look like this by time $t = 5$:

$$\begin{array}{ccccccccc} (0, 0) & \xrightarrow{r} & (1, 0) & \xrightarrow{u} & (1, 1) & \xrightarrow{u} & (1, 2) & \xrightarrow{u} & (1, 3) & \xrightarrow{r} & (2, 3) \\ t = 0 & & t = 1 & & t = 2 & & t = 3 & & t = 4 & & t = 5 \end{array}$$