Math 3215, Homework 5, Fall 2011

October 31, 2011

1. Let the joint pdf of X and Y be

   $$f(x, y) = 1/4, \ (x, y) \in S = \{(0, 0), (1, 1), (1, -1), (2, 0)\}.$$

   a. Are X and Y independent?

   b. Calculate $Cov(X, Y)$ and find ρ.

2. Let W equal the weight of laundry soap in a 1-kilogram box that is distributed in Southeast Asia. Suppose that $P(W < 1) = 0.2$ and $P(W > 1.072) = 0.08$. Call a box of soap light, good, or heavy depending on whether $\{W < 1\}$, $\{1 \leq W \leq 1.072\}$, or $\{W > 1.072\}$, respectively. In $n = 50$ independent observations of these boxes, let X equal the total number of light boxes and Y the number of good boxes.

   a. What is the jpdf of X and Y?

   b. Give the name of the distribution of Y along with the values of the parameters of this distribution.

   c. Given that $X = 3$, how is Y distributed conditionally (i.e., find $g(y|x = 3)$, the conditional pdf).

   d. Determine $E(Y|X = 3)$.

   e. Find ρ, the correlation coefficient of X and Y.

3. Prove that if X and Y are Poisson random variables, having parameters $\lambda_1 > 0$ and $\lambda_2 > 0$, respectively, then the random variable $Z = X + Y$ is Poisson with parameter $\lambda_1 + \lambda_2$.

4. If X is an $N(650, 625)$ random variable (normal with mean 650 and variance 625), find

   a. $P(600 \leq X < 660)$.

   b. A constant $c > 0$ such that $P(|X - 650| \leq c) = 0.9544$.

5. Let $Y = X_1 + \cdots + X_{15}$ be the sum of a random sample of size 15 from the distribution whose pdf if $f(x) = (3/2)x^2$, $-1 < x < 1$. Using the Central Limit Theorem, approximate

$$P(-0.3 \leq Y \leq 1.5).$$

6. Suppose that $Z = (X_1^2 + X_2^2 + X_3^2)^{1/2}$, where the X_i's are independent $N(0, 1)$ random variables. Determine $P(Z \geq 2)$.