Homework 6, Math 3215 – PROBLEMS NOT TO BE TURNED IN, BUT WORKED ON YOUR OWN

December 1, 2011

1. The distributions of incomes in two cities follow two Pareto type pdf's:

   $$f(x) = 2/x^3,\ 1 < x < \infty,\ \text{and}\ g(y) = 3/y^4,\ 1 < y < \infty.$$

   Here one unit represents 20,000 dollars. One person with income is selected at random from each city and let X and Y be their respective incomes. Compute $P(X < Y)$.

   Comment: Ok, so this is basically a 2D random variable problem. You will need to find the jpdf $h(x, y)$, and then to integrate over the given region.

2. Three components are placed in series. The time in hours to failure of each has the pdf

   $$f(x) \ = \ xe^{-x/500}/500^2,\ 0 < x < \infty.$$

   Since they are in series, we are concerned with the minimum time Y to failure of the three. Assuming independence, find the cdf and the pdf for Y and compute $P(Y \leq 300)$.

   Comment: This problem is kind of like that problem from the Maximum Likelihood Estimate and Unbiased Estimator notes about esimating the length t of the interval $[0, t]$ by choosing numbers at random from the interval. The idea is to convert the the problem at hand into a probability calculation involving three independent events.

3. Bowl A contains 100 red balls and 200 white balls; bowl B contains 200 red balls and 100 white balls. let p denote the probability of drawing a red ball from a bowl, but say that p is unknown, since it is unknown whether bowl A or bowl B is being used. We shall test the simple null hypothesis $H_0 : p = 1/3$ against the simple alternate hypothesis $H_a : p = 2/3$. Draw three balls at random, one at a time and with replacement from the selected bowl. Let X equal the number of red balls drawn. Then let the critical region (what we call the 'rejection region') be $C = \{x : x = 2, 3\}$. What are the values of α and β, the probabilities of Type I and Type II errors, respectively?

4. It was claimed that 75% of all dentists recommend a certain brand of gum for their gum-chewing patients. A consumer group doubted this claim and decided to test $H_0 : p = 0.75$ against the alternative hypothesis $H_1 : p < 0.75$, where p is the proportion of dentists who recommend this brand of gum. A survey of 390 dentists found that 273 recommended this brand of gum. Which hypothesis would you accept if the significance level is

   a. $\alpha = 0.05$?

   b. $\alpha = 0.01$?

   c. Find the p-value for this test.

5. Let X equal the thickness of spearmint gum manufactured for vending machines. Assume that the distribution of X is $N(\mu, \sigma^2)$. The target thickness is 7.5 hundredths of an inch. We shall test the null hypothesis $H_0 : \mu = 7.5$ against a two-sided alternate hypothesis using 10 observations.

   a. Define the test statistic and rejectioin region for an $\alpha = 0.05$ significance level. Sketch a figure illustrating this region.

   b. Calculate the value of the test statistic and clearly give your decision using the following $n = 10$ thickness in hundredths of an inch for pieces of gum that were selected randomly from the production line:

   $$7.65,\ 7.60,\ 7.65,\ 7.70,\ 7.55,\ 7.55,\ 7.40,\ 7.40,\ 7.50,\ 7.50.$$

   c. Is $\mu = 7.50$ contained in a 95% confidence interval for μ?

6. A 1-pound bag of candy-coated chocolate-covered peanuts contained 224 pieces of candy colored brown, green, and yellow. Test the null hypothesis that the machine filling these bags treats the four colors of candy equally likely; that is, test

$$H_0 \ : \ p_B = p_O = p_G = p_Y = 1/4.$$

The observed values were 42 brown, 64 orance, 53 green, and 65 yellow. You may select the significance level or give the approximate p-value.

Comment: Obviously you want to use a χ^2 test.

7. While testing a used tape for bad records, a computer operator counted the number of flaws per 100 feet of tape. Let X equal this random variable. Test the null hypothesis that X has a Poisson distribution with a mean of $\lambda = 2.4$ given that 40 observations of X yielded 5 zeros, 7 ones, 12 two, 9 threes, 5 fours, 1 five and 1 six. Let $\alpha = 0.05$.

   Hint: Combine five and six into one set; that is, the last set would be all x values ≥ 5.

8. A random sample $X_1, ..., X_n$ of size n is taken from a Poisson distribution with a mean of λ, $0 < \lambda < \infty$.

   a. Show that the maximum likelihood estimator for λ is $\hat{\lambda} = \overline{X}$.

   b. Let X equal the number of flaws per 100 feet of a used computer tape. Assume that X has a Poisson distribution with a mean of λ. If 40 observations of X yielded 5 zeros, 7 ones, 9 threes, 5 fours, 1 five, and 1 six, find the MLE estimate for λ.

9. Let $X_1, ..., X_n$ be a random sample of size n from the exponential distribution whose pdf is $f(x;\theta) = (1/\theta)e^{-x/\theta}$, $0 < x < \infty$, $0 < \theta < \infty$.

   a. Show that $\overline{X}$ is an unbiased estimator of θ.

   b. Show that the variance of $\overline{X}$ is θ^2/n.

   c. What is a good estimate of θ if a random sample of size 5 yielded the sample values 3.5, 8.1, 0.9, 4.4, and 0.5?