Study Sheet for Midterm 2, Math 3215, Fall 2011

November 15, 2011

First let me say that the exam will consist of 5 questions (the usual), but this time there will only be 1 proof; so, there will be 1 definition question, 1 proof question, and 3 computation questions. Obviously you are allowed (encouraged) to bring non-programmable calculators to the exam. Here are the topics I expect you to know for the exam:

1. Know how to find the pdf of a function of a random variable. Say, $Y = g(X)$, and say you know the pdf for X, then you want to be able to find the pdf for Y. (Memorizing a formula is not a good idea here – I could easily produce an "implicit analogue" of the problem where formulas are of no use). The right way to do it is to first find the cdf for Y, and then take its derivative.

2. Know the proof that $f(x) = e^{-x^2/2}/\sqrt{2\pi}$ is a pdf.

3. Be familiar with Gamma random variables and their connection to Poisson processes. Be able to work problems if I give you the pdf of a Gamma r.v. (you won't be expected to remember the pdf). Know the definition of the gamma function $\Gamma(x)$ and be able to prove basic facts, such as that $\Gamma(1/2) = \sqrt{\pi}$, $\Gamma(x+1) = x\Gamma(x)$, and $\Gamma(x) = (x-1)!$.

4. Know how to compute the moment generating function of several common random variables if I give you the pdf. For example, know how to find the mgf for normal r.v.'s, Poisson r.v.'s, uniform r.v.'s, and Gamma r.v.'s (given the pdf).

5. Know about joint pdf's (jpdf) for multidimensional r.v.'s $(X_1, ..., X_k)$. Know what it means for r.v.'s to be independent in terms of what their jpdf looks like. Know the fact that a binomial r.v. has the same distribution as the sum $X_1 + \cdots + X_k$ of k independent Bernoulli r.v.'s. Know how to find the marginal pdf of a given r.v.

6. Know how to find the pdf for $Z = \sqrt{X^2 + Y^2}$, given that X and Y are independent $N(0, 1)$ r.v.'s (basically, Z has a Rayleigh distribution). Know how to prove that if X and Y are independent Poisson, with parameters λ_1 and λ_2, respectively, then $X + Y$ is Poisson with parameter $\lambda_1 + \lambda_2$.

7. Know about conditional pdf's $f(x|y = c)$. Know about conditional expectation $E(X|Y = c)$. Know the Tower Property of Expectation $E_y(E_x(X|Y = y)) = E(X)$.

8. Know the fact that for r.v.'s $X_1, ..., X_k$ we have $E(X_1 + \cdots + X_k) = E(X_1) + \cdots + E(X_k)$. Note that this does not even assume the X_i's are independent. Know the fact that if $X_1, ..., X_k$ are *pairwise* independent, then $V(X_1 + \cdots + X_k) = V(X_1) + \cdots + V(X_k)$. Be able to use these facts to give quick proofs that the expectation and variance of a binomial r.v. with parameters n and p, are np and $np(1 - p)$, respectively.

9. Know the definition of the correlation coefficient – both the r.v. version and the stats version. Know some of its basic properties, such as $-1 \leq \rho \leq +1$, and $\rho = \pm 1$ if and only of (X, Y) are linearly related. Know that if X and Y are independent, then $\rho = 0$; and know examples where $\rho = 0$ but that X and Y are not independent.

10. Know that if $X_1, ...X_k$ are independent then $E(X_1 \cdots X_k) = E(X_1) \cdots E(X_k)$. This doesn't necessarily hold if the X_i's have some dependency (can you think of an example? Hint: What if all the X_i's are ± 1, and X_k is chosen to be the product $X_1 \cdots X_{k-1}$?).

11. Know the following VERY useful property of mgf's: if $X_1, ..., X_k$ are independent, then $M_{X_1 + \cdots + X_k}(t) = M_{X_1}(t) \cdots M_{X_k}(t)$. Know the fact that if $M_X(t) = M_Y(t)$ for all $t \in (-\epsilon, \epsilon)$ (for some $\epsilon > 0$), then $X \sim Y$. Know the notation $X \sim Y$ – it means that the cdf's of X and Y are the same.

12. Know the Law of Large Numbers and the Central Limit Theorem. Know how to use Chebyshev's inequality to prove the Law of Large Numbers.

13. Know the fact that the sum of k independent normal r.v.'s is a normal r.v. (those normals need not have the same mean and variance). Know how to use this fact to solve the "pedagogical case" of confidence intervals: namely, if you are given some random source X, known to be normal with unknown mean μ and known variance σ^2, then upon sampling k independent values $X_1, ..., X_k$ from the source you can determine a 95 percent confidence interval for μ. Be able to do the non-pedagogical case given the appropriate lookup tables and theorem linking the χ^2 and student-t r.v.'s to confidence intervals.

    You will not need to know the stuff on Maximum Likelihood Estimates and hypothesis testing for this exam – that will be material for the final.