To be published in Optics Letters:

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Accepted: 28 September 2007

Posted: 1 October 2007

Doc. ID: 86280

A New Model for Efficient Simulation of Spatially Incoherent Light Using the Wiener Chaos Expansion Method

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Abstract

We demonstrate a new and efficient technique for modeling and simulation of spatially incoherent sources, using the Wiener chaos expansion (WCE) method. By implementing this new model, we show that a practical-size photonic structure with a spatially incoherent input source can be analyzed more than two orders of magnitude faster compared to the conventional models without sacrificing the accuracy.

Many biological and environmental sensing applications demand spectral analysis of diffuse (i.e., spatially incoherent) light [1]. Since the diffuse optical signals are usually very faint and have a wide angular extent, sensing them using conventional spectrometers is not efficient. To improve the sensitivity of the optical spectrometers for diffuse light spectroscopy, a new class of spectrometers called multimodal multiplex spectrometers (MMSs), have been recently proposed and implemented using photonic crystals (PCs) [2] and volume holograms [3].

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While there have been a lot of recent efforts in the experimental development of MMSs, a parallel progress in the development of design and analysis tools, especially for more complicated material systems like PCs, is still missing. The analysis of the propagation of spatially incoherent light in PC structures requires detailed numerical simulation as no analytic representation of electromagnetic waves in such structures exists.

In this letter, we demonstrate the most efficient and accurate technique to date for modeling and simulation of spatially incoherent sources. While the model is quite general and can be implemented with different numerical simulation techniques and can be applied to any material system, we will implement our model using the finite difference time domain (FDTD) technique [4] for two-dimensional (2D) PC structures. The choice of 2D PCs is due to the importance of these structures for the development of on-chip spectrometers [5] and wavelength demultiplexers [6].

Figure 1 shows our simulation platform, which is composed of a 2D square lattice of air holes etched in silicon. The radius of the air holes is 0.3*a*, where *a* is the lattice constant. The source line is placed in front of the PC along line *A* and the electric field values are monitored along the output line *B*. All the input sources are excited with a TE polarization (where the electric field is parallel to the *z*-axis). The electromagnetic wave propagation throughout the structure is governed by the 2D Helmholtz wave equation

$$\nabla^2 E_z(x,y,t) - \mu\varepsilon(x,y)\frac{\partial^2 E_z(x,y,t)}{\partial t^2} = \mu\frac{\partial J_z(x,y,t)}{\partial t}, \qquad (1)$$

where the current density (J_z) is the source of excitation, and μ and ε are permeability and permittivity of the structure, respectively. Here our source is modeled as a one-dimensional array of spatially incoherent point sources along line *A*. For modeling the spatially incoherent source, any two point sources on line *A* should radiate independently of each other. This definition by itself can be used as the brute-force technique for numerical modeling of the spatially incoherent source. In brute-force modeling, we enforce zero correlation between the contributions from every two input point sources by separately analyzing the structure with each point source and adding the individual contributions at the output line *B* incoherently (i.e., in power) [7]. While this technique models the incoherent source perfectly, it is very time-consuming since it requires one simulation of the entire structure for each input point source. Therefore, the use of the brute-force technique is not a reasonable option and we use this technique only as a reference to assess the accuracy of our more efficient (and possibly approximate) technique.

To reduce the simulation time, we propose a new technique using the Wiener chaos expansion (WCE) method [8] to model the spatially incoherent source. Note that the input source along line *A* in Figure 1 is a deterministic function of time and its stochastic nature is only in the spatial dimension (i.e., y in Figure 1). To model the spatially incoherent source, we use the white noise, i.e., the derivative of the Brownian motion, to model the spatial part of the current density (J_z) in Equation (1). More precisely, we represent the spatially incoherent source along line *A* (i.e., $x = x_A$) as

$$J_z(y,t) = dW(y)V(t)\,, \tag{2}$$

where $V(t)$ is a deterministic function representing the time variation of the source and $dW(y)$ is the derivative of the Brownian motion representing the independent spatial randomness along y. Note that assuming J_z to be a separable function of space (y) and time (t) is consistent with all practical applications in which the time-variation of the source is assigned by the frequency range of operation and is usually the same at all points along the source line.

According to the WCE theorem [8], by choosing any orthonormal basis functions ($m_i(y)$), we can introduce a set of independent standard Gaussian random variables (ξ_i) such that

$$dW(y) = \sum_i \xi_i m_i(y), \tag{3}$$

where

$$\xi_i = \int_0^y m_i(s) dW(s) \qquad i = 1, 2, \ldots. \tag{4}$$

The WCE method separates the deterministic effects from the randomness (covered by ξ_i). Therefore, the original stochastic Helmholtz wave equation is reduced into an associated set of deterministic equations for the expansion coefficients. It can be shown that all the statistical moments of the random solutions of the original stochastic equation at the output line B in Figure 1 can be directly calculated using these expansion coefficients [8]. Obviously, by choosing the number of the expansion coefficients considered in Equation (3), the accuracy and the gain in the simulation time can be varied. Fortunately, it is known that WCE is a very fast converging expansion technique [8], and usually does not require many expansion coefficients. Thus, by using only a few terms in Equation (3), we can achieve enough accuracy in a very fast simulation for almost all practical optical structures.

Using the formulation described above, we need to solve the following set of deterministic equations

$$\nabla^2 E_{zi}(x,y,t) - \mu\varepsilon(x,y)\frac{\partial^2 E_{zi}(x,y,t)}{\partial t^2} = \mu m_i(y)\frac{dV(t)}{dt} \qquad i = 1,2,\ldots, \tag{5}$$

for the expansion coefficients ($E_{zi}(x,y,t)$). In the rest of this paper, we will discuss the simulation results obtained by solving the set of deterministic equations in Equation (5) using the FDTD technique.

For the numerical simulation, we choose a commonly-used sinusoidal modulated Gaussian pulse for the time function $V(t)$ [4],

$$V(t) = \sin(\omega(t - t_0))\exp\left(-\left(\frac{t - t_0}{T}\right)^2\right), \tag{6}$$

to cover a reasonable range of frequencies. We also choose a set of sinusoidal basis functions for $m_i(y)$ given by [8]

$$m_1(y) = \frac{1}{\sqrt{y_f}}, \tag{7-a}$$

$$m_i(y) = \sqrt{\frac{2}{y_f}}\cos\left[(i-1)\pi\frac{y}{y_f}\right] \qquad i = 2,3,\ldots, \tag{7-b}$$

where y_f is the total length of the input line A as shown in Figure 1. It is worth mentioning that in general we can choose any orthonormal basis for the spatial function [$dW(y)$ in Equation (3)]. The functions used in Equations (7) are primarily selected for their simplicity.

For the wave propagation simulation, we assume a PC structure (shown in Figure 1) with dimensions $x_f = 10a$ and $y_f = 20a$. The x-y plane is discretized so we get 24 grid cells per lattice constant (a) along both x and y axes. In the numerical simulation, we need to simulate the structure for each basis function $m_i(y)$ (i = 1, 2, ..., M) to find the corresponding $E_{zi}(x, y, t)$ defined in Equation (5) at the output line B (i.e., $x = x_B$). We can then calculate all statistical properties of the output field using these corresponding expansion coefficients. For example, prior to calculate the power spectrum of light at the output line B, we need to find the second moment [7] of the random field values (i.e., $\langle E_z^{\,2}(x, y, t)\rangle_e$) in Equation (1) which can be simply calculated using the corresponding expansion coefficients ($E_{zi}(x, y, t)$) as [8]

$$\langle E_z^{\,2}(x, y, t)\rangle_e = \sum_i \left|E_{zi}(x, y, t)\right|^2 . \tag{8}$$

The key advantage of the WCE technique is its fast convergence. With M expansion coefficients selected in Equation (5), the total simulation time is M times the simulation time of the original structure with a deterministic input. For the PC structure in Figure 1, we need $T_s = 2^{16} = 65536$ time steps to get steady state results at the output line B.

The simulation result of the electric field power spectrum versus the normalized frequency at a typical point on the output line B is shown in Figure 2. For this simulation, we used only $M = 15$ expansion coefficients. The same data calculated using the brute-force technique is also shown in Figure 2 for comparison. The excellent agreement between the fast simulation using the WCE model and the long simulation using the brute-force model is visible from Figure 2.

To calculate the gain in the simulation time using the WCE model, we just need to compare the total number of simulations of the entire structure needed in the two models. This number is equal to $M = 15$ (i.e., the number of the expansion coefficients) for the WCE model while it is equal to the number of FDTD grid cells along the source line A, which is $20 \times 24 = 480$ (corresponding to $y_f = 20a$ and 24 grid cells per lattice constant, *a*). Thus, the simulation based on the WCE model is 32 times faster than that using the brute-force model. For larger structures that are needed in practical applications, this advantage is higher (at least two orders of magnitude).

Figure 3 shows the relative error of the WCE model with respect to the brute-force model as a function of the number of expansion coefficients, M. To calculate the relative error, we first calculate the sum of the square of the differences between the two power spectra (from the two models) for all frequencies and all points at the output line B. Then, we divide this sum by the sum of the square of the power spectrum for all frequencies and all points at the output line B calculated using the brute-force model. Note that we use all the frequencies and all the points at the output line B to show the accuracy of our model. Figure 3 clearly shows that the results of the WCE model very quickly become close to those of the brute-force model with negligible error (the error is 0.08% for M = 15), which is a direct observation of its fast convergence. Moreover, the results obtained by considering the first M = 15 expansion coefficients for the structure in Figure 1 are accurate enough for almost all practical applications.

In summary, we demonstrated here a new and efficient technique for accurate modeling and simulation of a diffuse (or spatially incoherent) light source with at least two orders of magnitude improvement in simulation time compared to the conventional brute-force technique. This gain in the simulation time is achieved without sacrificing the accuracy. The proposed technique is quite general and can be used for any structure with any material system.

Acknowledgment

This work was supported by Office of Naval Research under Contract No. N00014-05-0303 (M. Spector).

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Figure captions

Figure 1. The schematic of a 2D square lattice PC structure of air holes in silicon with hole radius *r* and lattice constant *a*. The input (or source) and output lines are shown by *A* and *B*, respectively.

Figure 2. (Color online) The electric field power spectrum as a function of normalized frequency at a typical point on the output line *B* in Figure 1. The simulation result of WCE model was obtained with only $M = 15$ expansion coefficients.

Figure 3. The percentage relative error of the WCE model with respect to the brute-force model as a function of the number of expansion coefficients (M).

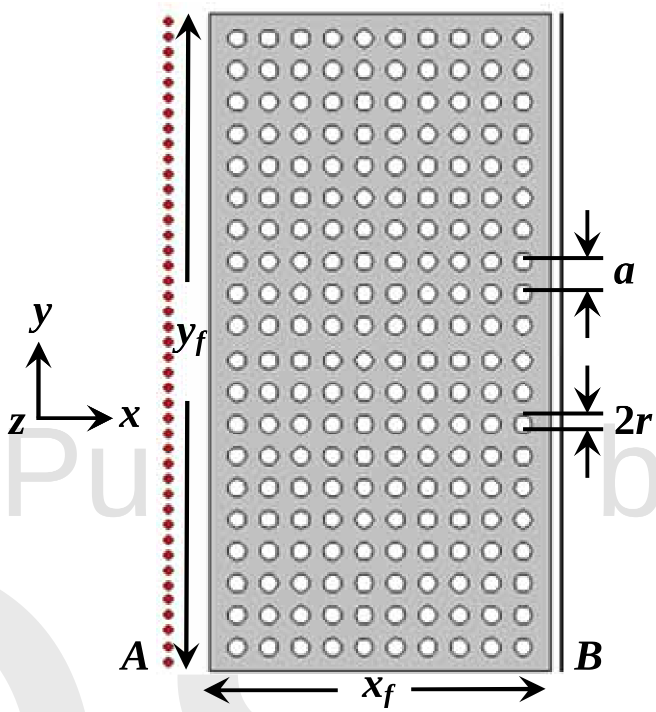

Figure 1

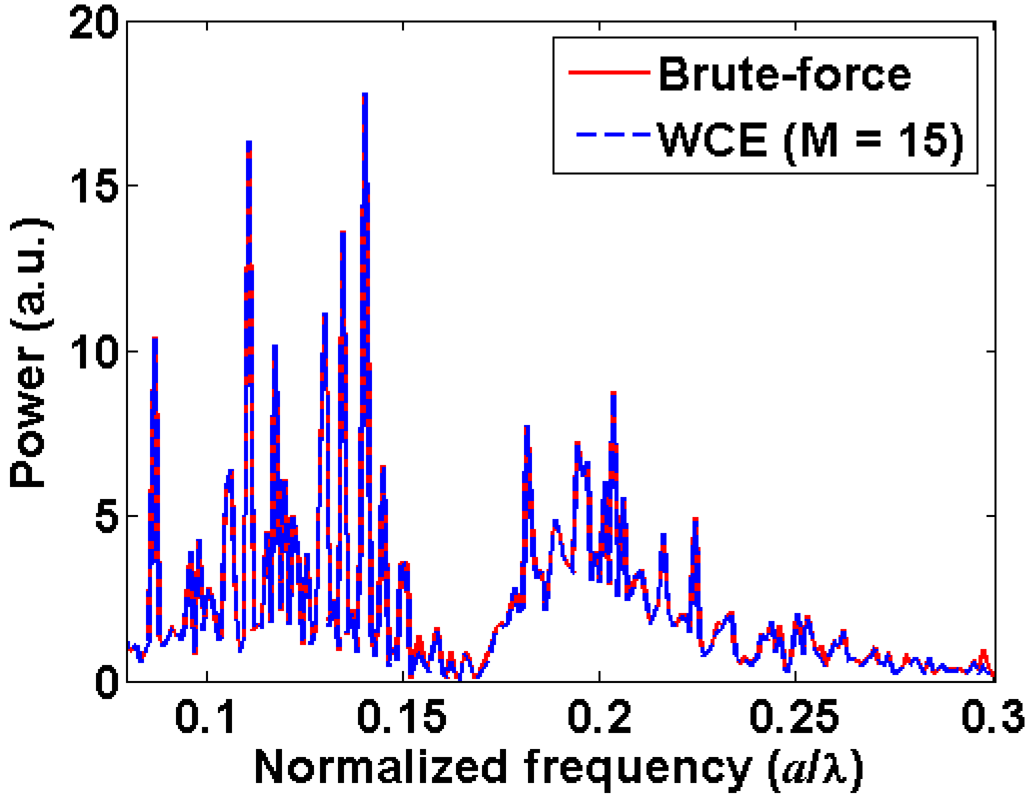
20
15
10
5
0
Power (a.u.)
0.1
0.15
0.2
0.25
0.3
Normalized frequency (a/λ)
Brute-force
WCE (M = 15)

Figure 2

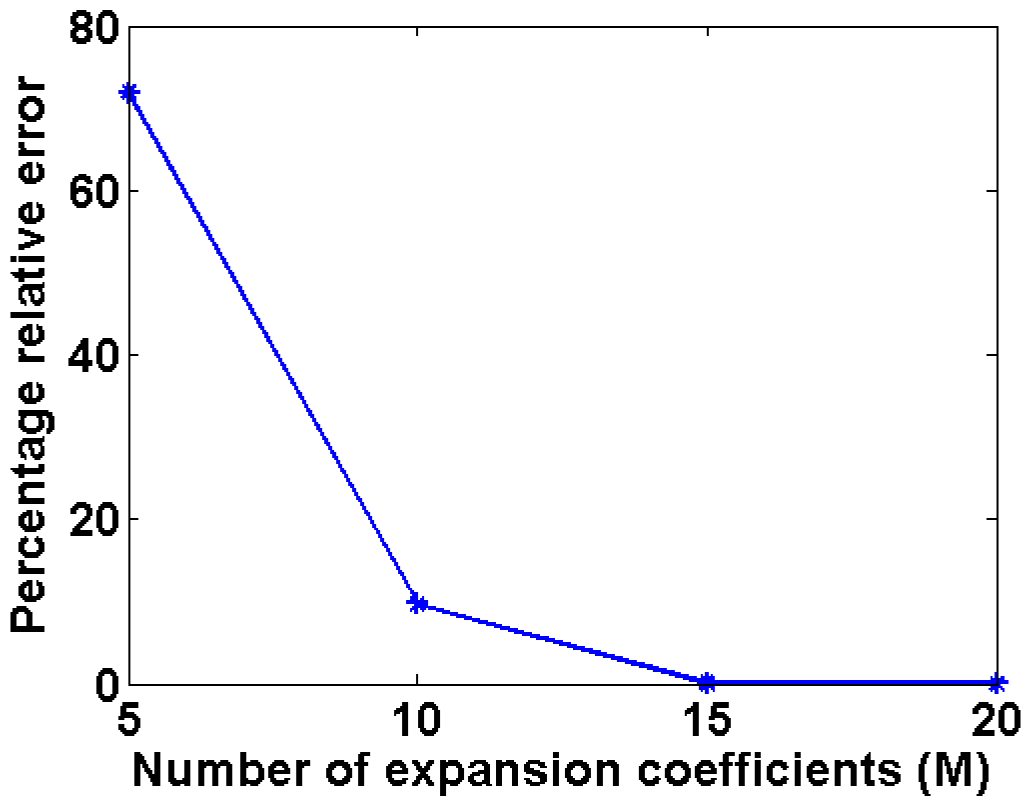
80
60
40
20
0
5
10
15
20
Percentage relative error
Number of expansion coefficients (M)

Figure 3