Solutions for Quiz 1 for Calculus ++, Math 2605B1-2, August 26, 2004

Problem I: Equation for the plane P_1:

$$x + y + z = 3\ .$$

Row reducing the system

$$x + 2y + 3z = 6\ ,\ x + y + z = 3\ ,$$

yields

$$\mathbf{x}(t) = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Problem II: The point

$$\mathbf{x_0} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$$

is on the plane. To find the distance we have to split the vector $\mathbf{p_2} - \mathbf{x_0}$ in a component parallel to the vector normal to the plane and one that is perpendicular to the plane. The component along the normal vector is what we are looking for. The normal unit vector is

$$\mathbf{u} = \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\ ,$$

$$\mathbf{p_2} - \mathbf{x_0} = \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}\ .$$

The component parallel to $\mathbf{u}$ is

$$\mathbf{u} \cdot (\mathbf{p_2} - \mathbf{x_0})\mathbf{u} = -\frac{1}{14} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

and the distance is given by the length of this vector which is

$$\frac{1}{\sqrt{14}}\ .$$

Problem III: The line is given (in parametrized form) by

$$\mathbf{x}(t) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

The base point is the vector

$$\mathbf{x_0} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} .$$

We have to find the component of $\mathbf{p_3} - \mathbf{x_0}$ which is perpendicular to the direction vector

$$\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} ,$$

which, when normalized takes the form

$$\mathbf{u} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} .$$

This component is given by

$$(\mathbf{p_3} - \mathbf{x_0}) - \mathbf{u} \cdot (\mathbf{p_3} - \mathbf{x_0})\mathbf{u}$$

which equals

$$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} .$$

The distance is the length of this vector which is $\frac{\sqrt{6}}{2}$.

Extra credit: The point

$$\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

is a base point for this plane. The direction vector of the line

$$\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$$

must be in this plane as must be the vector pointing from the base point to $\mathbf{p_1}$ which is given by

$$\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

The cross product of the two vector sin the plane yields the normal vector which is $[1, 1, 1]$. This plane is given by

$$x + y + z = 3 .$$