PRACTICE FINAL EXAM, MATH 3406 K, APRIL 20, 2020

Print Name:

This test is open books and open notes. By writing in longhand "I abide by the Georgia Tech honor code' and signing it you promise not to seek any help from other people nor do you seek the solutions from other sources such as the internet. You have a 48 period during which you can take the final exam. Once you start you have 4 hours to finish it. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. State your work clearly, otherwise credit cannot be given. Likewise, write legibly!

Your statement and your signature:

Problem 1: Find an example of a 3×5 matrix which is in reduced echelon form and has variables 2 and 4 free.

Problem 2: Write an example of a 3×2 matrix that has the plane $x + y + z = 0$ as its column space.

Problem 3: Factorize the symmetric matrix

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

in the form LDL^T where L is lower triangular with ones on the diagonal and D is diagonal.

Problem 4: Consider the matrix

$$\begin{bmatrix} 1 & 0 & 2 & -3 \\ 2 & 6 & -2 & 12 \\ 2 & 3 & 1 & 3 \end{bmatrix}$$

a) Find a basis for the column space $C(A)$
b) Find a basis for $N(A)$
c) For $C(A^T)$
d) For $N(A^T)$.

Problem 5: The row space of a matrix is spanned by the vectors

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix}$$

find a basis for the Null space of the matrix.

Problem 6: Consider the subspace S of $\mathbb{R}^4$ given by all the vectors that are perpendicular to the vectors

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}.$$

Find the unique vectors $\vec{x} \in S, \vec{y} \in S^{\perp}$ such that

$$\vec{x} + \vec{y} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

Problem 7: Given S a three dimensional subspace of $\mathbb{R}^4$, consider the linear transformation that reflects any vector $\vec{x}$ about S. Find a basis such that the matrix associated with H in that basis is as simple as possible.

Problem 8: Does there exist a matrix A such that $A + cI$ is invertible for all $c \in \mathbb{C}$?

Problem 9: Consider the Markov matrix

$$M = \begin{bmatrix} 4/5 & 2/5 \\ 1/5 & 3/5 \end{bmatrix}.$$

Compute the steady state, i.e., compute $\lim_{n\to\infty} M^n$.

Problem 10: Compute the eigenvalues of the Hermitean matrix

$$A = \begin{bmatrix} 7 & -4i \\ 4i & 1 \end{bmatrix}$$

b) Find a unitary matrix U such that U^*AU is diagonal.

Problem 11 : Solve $\frac{du}{dt} = Au$ where $A = \begin{bmatrix} 0 & 4 \\ 0.25 & 0 \end{bmatrix}$, starting from $u(0) = (100, 100)$.

Problem 12: Compute the singular value decomposition for the matrix

$$A = \begin{bmatrix} 0 & 6 & 3 \\ 4 & 2 & 5 \end{bmatrix}$$

Check your answer!

Problem 13 : (3 points each) True or False

a) An invertible matrix can be similar to a singular matrix.

b) Every square matrix A is of the form $A = UTU^*$ where T is upper triangular and U is unitary.

c) Let S be a five dimensional subspace of $\mathbb{R}^6$. Every basis of $\mathbb{R}^6$ can be reduced to a basis of S by removing one vector.

d) If A and B are positive definite, so is $A + B$.

e) If A is a square matrix with $A^T = -A$, then $A\vec{x}$ is perpendicular to $\vec{x}$ for any $\vec{x}$.