

HOMEWORK 1 , DUE THURSDAY JANUARY 16

Problem 1, (5 points): The symmetric difference of two sets A, B is defined to be

$$A\Delta B := (A \setminus B) \cup (B \setminus A) .$$

Show that

$$A\Delta B = (A \cup B) \setminus (A \cap B) .$$

Problem 2, (5 points): Let $f : X \rightarrow Y$ be a function between two sets X, Y . For any two sets $A, B \subset Y$ show that

$$f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B) , \quad f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$$

Problem 3, (5 points): With the same assumptions as in the previous problem, is it true that for any two sets $A, B \subset X$, $f(A \cup B) = f(A) \cup f(B)$? Is it true that $f(A \cap B) = f(A) \cap f(B)$?

Problem 4, (7 points): Recall that a metric space X is compact if and only if every open cover of X has a finite sub-cover. Prove that any sequence in a compact metric space has a convergent sub-sequence.

Problem 5, (3 points): Prove that every compact metric space is complete.