HOMEWORK 2, DUE THURSDAY JANUARY 23

Problem 1, (5 points): Consider the set X of rational numbers in the interval $[0,1]$. For any of the intervals $(a,b), [a,b], (a,b], [a,b) \subset [0,1] \cap \mathbb{Q}$ define its measure to be

$$m(a,b) = b - a \ .$$

Show that this measure cannot be extended to a σ additive measure on this set.

Problem 2, (5 points): Recall that the symmetric difference of two sets $A, B \subset \mathbb{R}^d$ is given by $A \Delta B = (A \setminus B) \cup (B \setminus A)$. Prove that

$$||A|_e - |B|_e| \leq |A \Delta B| \ .$$

Problem 3, (5 points): Let $\{E_k\}_{k=1}^{\infty}$be a sequence of sets in $\mathbb{R}^d$. Define the sets

$$\limsup_{k\to\infty} E_k := \cap_{j=1}^{\infty} \left(\cup_{k=j}^{\infty} E_k \right) \ \text{ and } \ \liminf_{k\to\infty} E_k := \cup_{j=1}^{\infty} \left(\cap_{k=j}^{\infty} E_k \right)$$

Show that $\limsup_{k\to\infty} E_k$ consists of those points $x \in \mathbb{R}^d$ that belong to infinitely many of the E_k and that $\liminf_{k\to\infty} E_k$ consists of the points $x \in \mathbb{R}^d$ that belong to all but finitely many of the E_k.

Problem 4, (5 points): Please do problem 2.1.39 in Heil's book.

Problem 5, (5 points): Please do problem 2.1.40 in Heil's book.