Errata for the Second Edition of "Analysis", complete as of July 1, 2006

We thank the many friends and colleagues who took the trouble to tell us about errors and missprints in the second edition.

On page 13, the notion of strict positivity is not correctly defined. A measurable function is strictly positive on a measurable set A if there exists $\varepsilon > 0$ such that the set $\{x \in A : f(x) < \varepsilon\}$ has measure zero. This notion is used on page 281 in the proof of Theorem 11.8.

On page 26 in Theorem 1.13 one has to assume that the measure space (Ω, Σ, μ) is sigma finite. Only then can one apply Fubini's theorem.

On page 28 in Theorem 1.14 (Ω, Σ, μ) has to be sigma finite since the proof relies on the layer cake representation (see the erratum concerning page 26). Also on page 28, in Theorem 1.14 (Bathtub Principle) it is necessary to add a a caveat to the uniqueness statement at the end. The assumption is needed that the infimum, I, in (1), is finite.

On page 48, displayed equation between (4) and (5): Replace $\lambda(y)$ by $\lambda(y)^p$.

On page 71 the line after the first displayed formula should read $\|f\|$ and not $\|f\|^2$.

On page 77, Exercise 2.23, add the sentence: Here, Ω is any Lebesgue measurable subset of R^n of positive measure (which need not contain any ball).

On page 81 in the line after formula (3) replace " ..two monotone functions.." by " ..two monotone non-decreasing functions.." . We also have to assume that $\phi_1(0)$ as well as $\phi_2(0)$ are zero, so that the layer cake representation Theorem 1.13 holds.

On page 81 in item (v) we have to require the function Φ to be lower semi-continuous since the left side is lower semi-continuous by definition. Also on page 81 just before (vi) change 'nonincreasing' to 'nondecreasing'.

On page 84 equation (2), note that $J'_+(t) \geq 0$ and hence all the expressions in formula (2) are properly defined.

On page 95, Exercise 3: A and B should be sets of finite measure.

On page 102 in the last line of equation (13) replace $\|g\|^q\|h\|^r$ by $\|g\|_q\|h\|_r$.

On page 107 after formula (5) it is mentioned that (3) and (5) are equivalent. The meaning of this statement is explained in Exercise 2 on page 121.

On page 121, Exercise 4.2: Change p to q.

On page 121, in Exercise 4.4, one has to assume that $\lambda > 0$.

On page 143, two lines before Section 6.9, the summation should be $j = 1$ to m and not $j = 1$ to n.

On page 152, top equation, replace $|\nabla f(x)|^2$ by $|\nabla f(x)|$, i.e., remove the square from the right side.

On page 158 in the last equation there should be a '(' before '∂G_y'.

On page 180 in the middle of the page, the strong limit exists in L^p for $p < \infty$.

On page 184 in the second displayed formula replace μ by ν, i.e., the formula should read

$$K_\nu(z) \approx \frac{1}{2}\Gamma(\nu)\left(\frac{1}{2}z\right)^{-\nu}$$

On page 194, in the displayed equation in the middle of the page just above "and hence", replace the '-i' in the rightmost term by ' +', i.e, it should read $\chi_m(\nabla + iA)f + (\nabla\chi_m)f$.

On page 196 in Exercise 7, the inequality is in the wrong direction.

On page 202, in the expression on the last line of the statement of Theorem 8.3, $(x-a)$ should be $|x-a|$.

On page 207 in the last integral of the second displayed equation: the lower limit should be a and not x.

On page 213, the inequality $0 < x_n < |x|\cos(\theta)$ in the second displayed expression is incorrect. It should be replaced by $x_n > |x|\cos(\theta)$.

On page 214, Theorem 8.9 (iii) is not correct. The supremum has to be taken in ω which is an open *bounded* subset of Ω.

On page 218 in the line after equation (2) replace g^α by g_α.

On page 218, in Theorem 8.11 the case $p = 1$ is included in the statement. The Theorem as it stands is correct, but the proof given does not work in the case $p = 1$ since Theorem 2.18 requires $p > 1$. To prove the Theorem for $p = 1$, we proceed as before, i.e., we assume that there exists a sequence of functions f^j such that $\|f^j - \int gf^j\|_q = 1$ for all j and that $\|\nabla f^j\|_1 \to 0$. Here q may be any number less than $n/(n-1)$ and hence $q > 1$. Next note that the sequence $h^j = f^j - \int gf^j$ has the same gradient as f^j and hence $\|\nabla h^j\|_1 \to 0$. By Theorem 2.18 there exists a function h in $L^q(\Omega)$ and a subsequence again denoted by h^j such that $h^j \rightharpoonup h$ weakly in $L^q(\Omega)$. For any function $\phi \in C_c^\infty(\Omega)$ we have that $\int h^j\nabla\phi = -\int \nabla h^j\phi$ and since $\|\nabla h^j\|_1 \to 0$, we learn that $\int h\nabla\phi = 0$ for all $\phi \in C_c^\infty(\Omega)$, i.e., $\nabla h = 0$ in the sense of distributions. Thus, by Theorem 6.11 h is constant. Clearly $0 = \lim_{j\to\infty}\int h^j g = \int hg$ and thus, $h = 0$. By the Rellich-Kondrachev theorem we know that the space $W^{1,1}(\Omega)$ is compactly embedded in $L^q(\Omega)$ for all $q < n/(n-1)$. (Note that this formulation of the Rellich-Konrachev is slightly more general then Theorem 8.9. For a proof, see e.g., Brézis.) Hence h^j converges strongly in $L^q(\Omega)$ and therefore $\|h\|_q = 1$ which is a contradiction.

On page 220 the constant in Nash's inequality is not correct. The factor

$$\left(1 + \frac{n}{2}\right)^{1+n/2}$$

should be replaced by

$$\left(1+\frac{n}{2}\right)^{1+2/n} .$$

On page 230 after equation (3) remove the ‘h’ from Yoshida. It should read Yosida.

On page 235, Exercise 1 is not correctly stated. Consider three dimensions and take as a domain Ω the whole space without the origin. Removing one point does not change the Sobolev space, i.e., $H_0^1(\Omega) = H^1(\mathcal{R}^3)$. This example shows that the second assertion in the exercise is not correct. If one changes the problem by considering open and bounded sets then the second assertion is true. In fact it suffices the consider sets Ω whose complement is a set of positive capacity (see Section 11.15).

On page 240 in the third line, replace ‘in (2) become’ by ‘in (3) become’.

On page 250 the right side of equation (5) should have an overall factor $[(n-2)|\mathbb{S}^{n-1}|]^{-1}$, i.e., it should read

$$V(x) = \left[(n-2)|\mathbb{S}^{n-1}|\right]^{-1}\left[|x|^{2-n}\int_{|y|\leq|x|}\mu(\mathrm{d}y) + \int_{|y|>|x|}|y|^{2-n}\mu(\mathrm{d}y)\right]$$

On page 259, after equation (4), replace “ ... K_{f_2} and K_{f_1} have the same continuity and differentiability properties.” by “... K_f and K_{f_1} have the same continuity and differentiability properties in B_1.”

On page 268 in the last equation on the page the limit sign replace $j \rightharpoonup \infty$ by $j \to \infty$.

On page 271, line 3, the potential $|x|^{-3}$ is a bad example since it is not locally integrable. Considering $|x|^{-5/2}$ will do the trick, since the kinetic energy scales with λ^2 and the potential scales with $\lambda^{5/2}$, thus driving the energy to $-\infty$.

On page 272 in equation (6) change $(\psi, V\psi)$ to $|(\psi, V\psi)|$.

On page 276, in Remark (2), replace C_0^∞ by C_c^∞.

On page 278, bottom, ”follows the integration by parts argument ...” should read ”follows the lines of the integration by parts argument ...”

On page 301 in the third version of the min-max principle, the function ϕ appearing in formula (5) should be normalized, $(\phi,\phi) = 1$.

On page 318 replace G_- by G_R in the first displayed equation.

On page 319, Eq. (2) in Thm. 12.9 a square is missing on the left side, namely $|\widetilde{\psi}(k,y)|^2$

On page 339 the reference to Hermann Weyl is not in alphabetical order.