ERRATA OF THE FIRST PRINTING

The statement in Exercise 1.22 on page 34:

Theorem 1.10 (product measure) asserts that f and g are measurable functions. Prove this by imitating the proof of the section property in Sect. 1.2.

should be replaced by:

Theorem 1.10 (product measure) asserts that f and g are measurable functions. Prove this by imitating the proof of the section property in Sect. 1.2 and by using the Monotone Class Theorem.

Problem 2.3 is the same as problem 1.15 and should be discarded.

On page 113, replace the words

Hermitian, positive–definite $n \times n$ matrix

by

symmetric $n \times n$ matrix whose real part is positive definite

On page 153, the third line from the bottom: replace

$\psi := 1$

by

$\psi \equiv 1$

On page 208 at the end of Remark (1) replace

$f(x) \leq F$.

by

$f(x) \leq F$, where F is now the supremum of f over the boundary of the domain Ω.

On page 218, just before equation (4), delete:

for $z \in B_{i+1}$

On page 225, sixth line from the top: replace χ by χ_{2R}.

On page 259, last paragraph:

Replace

Our next goal is to deduce properties c)–e) of ϕ, as well as $\phi \le 1$ on A. Item c) is proved as follows. Let η be any *nonnegative* function in $C_c^\infty(\mathbf{R}^n)$, in which case $\Delta\eta$ is also in $C_c^\infty(\mathbf{R}^n)$. For $\varepsilon > 0$ let $f_\varepsilon := f - \varepsilon g$ with $g = |x|^{1-n} * \Delta\eta$. Correspondingly,

$$\phi_\varepsilon := |x|^{1-n} * f_\varepsilon = \phi - \varepsilon(|x|^{1-n} * |x|^{1-n}) * \Delta\eta.$$

(We are using Fubini's theorem here to exchange the order of integration in the repeated convolution.) By Theorem 6.21 (solution of Poisson's equation) and the fact that $|x|^{1-n} * |x|^{1-n} = C_n|x|^{2-n}$ we have that

$$-(|x|^{1-n} * |x|^{1-n}) * \Delta\eta = C_n'\eta,$$

with $C_n' > 0$. Therefore, f_ε is an admissible function for every $\varepsilon > 0$, because $\phi_\varepsilon \ge \phi$. Since f was a minimizer, we can conclude that

by

Our next goal is to deduce properties c)–e) of ϕ, as well as $\phi \le 1$ on A. Item c) is proved as follows. Let η be any *nonnegative* function in $C_c^\infty(\mathbf{R}^n)$, in which case $\Delta\eta$ is also in $C_c^\infty(\mathbf{R}^n)$. For $\varepsilon > 0$ let $f_\varepsilon^j := F^j - \varepsilon g$ with $g = |x|^{1-n} * \Delta\eta$. Correspondingly, with $\Phi^j := |x|^{1-n} * F^j$,

$$\phi_\varepsilon^j := |x|^{1-n} * f_\varepsilon^j = \Phi^j - \varepsilon(|x|^{1-n} * |x|^{1-n}) * \Delta\eta.$$

(We are using Fubini's theorem here to exchange the order of integration in the repeated convolution.) By Theorem 6.21 (solution of Poisson's equation) and the fact that $|x|^{1-n} * |x|^{1-n} = C_n|x|^{2-n}$ we have that

$$-(|x|^{1-n} * |x|^{1-n}) * \Delta\eta = C_n'\eta,$$

with $C_n' > 0$. Since $\phi_\varepsilon^j \ge \Phi^j$, f_ε^j is an admissible function for every $\varepsilon > 0$ and all j. From (1) and $F^j \to f$ in $L^2(\mathbf{R}^n)$, we can conclude that

In Exercise 15 on page 262: (7) should be 11.15(7) (3 times) and (8) should be 11.15(8).

Errata as of May 2, 1997

On page 23 first line starting with PROOF. The statement

The measurability of f and g parallels the proof of the section property in Sect. 1.2 and is left to Exercise 22.

should be replaced by

The measurability of f and g parallels the proof of the section property in Sect. 1.2 and uses the Monotone Class Theorem; it is left to Exercise 22.

On page 29 in the proof of Theorem 1.15 second paragraph: Replace

Let E be any set in Ω and.....

by

Let E be any set in Ω with $\mu(E) < \infty$ and.... .

On page 31 in the penultimate paragraph just below formula (7) there should be no indent before 'and hence'.

On page 31 after formula (7) before

By setting $E = \Omega$,

insert the following new paragraph:

In case $\mu(E) = \infty$ equation (1) holds for any set A by (iii) and, in particular, for *any* union (countable or not) of sets. Equation (5) is trivial in case $\mu(E \cap \cup_{i=1}^m B_i) = \infty$. If $\mu(E \cap \cup_{i=1}^m B_i)$ is finite simply replace E by $E' := E \cap \cup_{i=1}^m B_i$ and then the case $\mu(E) < \infty$ applied to E' yields (5). Thus, (6) holds generally.

On page 31 second line in the last paragraph: delete the part starting with 'monotone class.' and ending with 'Hence Σ is a' It should read: ... that Σ is a sigma–algebra...

On page 59 at the beginning of the second paragraph: Replace the sentence

The bound (1) clearly shows that it suffices....
by
Clearly, it suffices ....

On the same page the next sentence delete
(1) again shows that

On page 60, line 10 from the top: Replace
By the layer cake representation, 1.13, and Fubini's theorem....
by
By the layer cake representation,1.13, Fubini's theorem and Fatou's lemma ....

On page 78 in the middle: Replace the formulas

$$= \int_R \int_R f_j(x) g_k(x-y) h_l(y-(a-b-c)t) \mathrm{d}x \mathrm{d}y$$

$$= \int_R \int_R u_{jk}(y) h_l(y-(a-b-c)t) \mathrm{d}y$$

by the formulas

$$= \int_R \int_R f_j(x) g_k(x-y) h_l(y+(a-b-c)t) \mathrm{d}x \mathrm{d}y$$

$$= \int_R u_{jk}(y) h_l(y+(a-b-c)t) \mathrm{d}y$$

On page 94 one line above the formula (13): Replace
We write a point $x \in R^n$
by
We write a point $x \in R^{n+m}$

On page 99 first line:Replace
There is an alternative expression,
by
For $q > 1$ there is an alternative expression,

On page 105 formula (13): Replace the part of the formula that reads

$$\mathcal{J}_{\mathcal{S}}(x) = \left(\frac{2}{1+|x|^2}\right)^2$$

by

$$\mathcal{J}_{\mathcal{S}}(x) = \left(\frac{2}{1+|x|^2}\right)^n$$

On page 112 at the end of section 4.7: Replace

$$(x) = ...$$

by

$$f(x) = ...$$

On page 163 second line in the last paragraph of Section 7.7: Replace
$\int \nabla u^j \cdot \nabla v$
by
$-\int \nabla u^j \cdot \nabla v$. (In other words, stick in a minus sign).

On page 167 last line in Section 7.10: Replace 7.10 by 7.9

On page 181 in Exercise 4: Replace $\frac{1}{t}$ by $\frac{1}{t^2}$

On page 187 fourth line after formula (5), delete the comma between the braces

On page 192 fourth line from the bottom: replace $f_1 - f_2$ by $f - g$

On page 239 towards the end of the section 11.4: close up 'uniformly' and 'on bounded...'

In the index, Hölder should come after Hilbert–space.

Errata as of May 5, 1997

On page 28 Eq.(2) replace
$... + c\chi_{\{f=s\}}$ by $... + c\chi_{\{f=s\}}(x)$

On page 87, problem 2. Replace
Show that every open subset of the real line is a disjoint union of open intervals.
by
Show that every open subset of the real line is a countable disjoint union of open intervals.

On page 102 Eq.(21) The first integral in the second line: Replace the integrand $....b^{m-1}\mathrm{d}b$ by $....b^{r-1}\mathrm{d}b$.

On page 120 the line just after Eq.(3). Replace
$\lambda^{-1/2}$ by $\lambda^{-n/2}$.

On page 192, in the line just above Eq.(2), delete the words 'vector–valued'

On page 193 Eq.(5). Replace $e^{t|p|}$ by $e^{-t|p|}$

On page 198 at the end of the first sentence after REMARKS: Replace
....differentiable
by
....differentiable (what does this mean, precisely?).

On page 199, replace the wording of Exercise 2 by the following one:

Suppose somebody tries to define $H_0^1(\Omega)$, $\Omega \in \mathbf{R}^n$ as the set of those functions in $H^1(\mathbf{R}^n)$ that vanish outside the set Ω. What difficulty would be encountered with such a definition? For each n give an example where this definition gives the right answer and one where it does not.

Hint. Consider $H_0^1(\Omega)$ where $\Omega = (-1,\ 1) \sim \{0\}$ and and describe all the functions in this space.

Errata as of May 12, 1997

On page 246 after formula (2): Replace
... nonpositive functions $f \in L^1_{loc}(\mathbf{R}^n)$ and $g \in L^2(\mathbf{R}^n)$.
by
... nonpositive function $f \in L^1_{loc}(\mathbf{R}^n)$ and some $g \in L^2(\mathbf{R}^n)$.

On page 253 in formula in the middle of page: Replace

$$|x|^{-1} \int_{|y| \leq |x|} \rho_N(x) \mathrm{d}x + \int_{|y| > |x|} |y|^{-1} \rho_N(x) \mathrm{d}x$$

by

$$|x|^{-1}\int_{|y|\leq|x|}\rho_N(y)\mathrm{d}y+\int_{|y|>|x|}|y|^{-1}\rho_N(y)\mathrm{d}y$$

On page 257 in the statement of Theorem 11.16 formula a): replace $\int_{\mathbf{R}^n} f^2$ by $C_n\int_{\mathbf{R}^n} f^2$.

Errata as of May 21, 1997

On page 191 the second formula from the bottom reads

$$\leq K(\|f\|_2^2+\|\nabla f\|_2^2)$$

It should be replaced by

$$\leq K(\|f\|_2^2+\|\nabla f\|_2^2)^{1/2}$$