

MATH 2551, Spring 2017
Practice Exam 1, Chapter 12 and 13

Guideline: Please read the following carefully.

Remember to show all your work; including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. One side of a sheet of paper (letter size) for formulae, calculator is NOT allowed in this exam. You have 50 minutes.

Problem 1. Calculations.

(a) $\frac{d}{dt}[(2t\mathbf{i} + \sqrt{t}\mathbf{j}) \bullet (t\mathbf{i} - 3\mathbf{j})]$

(b) $\frac{d}{dt}[(\cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}) \times (3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})]$

(c) $\frac{d}{dt}[e^{\cos 2t}\mathbf{i} + \ln(1 + t^2)\mathbf{j} + (1 - \cos t)\mathbf{k}]$

Problem 2 A golf ball is hit at time $t = 0$. Its position vector as a function of time is given by

$$\mathbf{r}(t) = 2t\mathbf{i} + 3t\mathbf{j} + (-t^2 + 4t)\mathbf{k}.$$

Notice that at $t = 0$ the ball is at the origin of the coordinate system. The xy plane represents the ground. At some time $t_1 > 0$ the ball will return to the xy plane at some point $P(a, b, 0)$.

(a) Compute the velocity, the acceleration and the speed of the ball at an arbitrary time t .

(b) Find the time $t_1 > 0$ and the coordinates of the point P where the ball hits the xy plane again.

(c) Set up a definite integral equal to the length of the arc of the trajectory from the origin to the point P . You do not have to evaluate the integral.

(d) Find the equation of the line tangent to the trajectory at P .

(e) Find the equation of the vertical plane containing the trajectory.

(f) Find the curvature of the trajectory at P .

Problem 3 At each point $P(x(t), y(t), z(t))$ of its motion, an object of mass m is subject to a force:

$\mathbf{F}(t) = m\pi^2[4\cos(\pi t)\mathbf{i} + 3\sin(\pi t)\mathbf{j}]$. Given that $\mathbf{v}(0) = -3\pi\mathbf{j} + \mathbf{k}$, and $\mathbf{r}(0) = 3\mathbf{j}$. find the following:

(a) The velocity $\mathbf{v}(1)$.

(b) The speed $v(1)$.

(c) The momentum $\mathbf{p}(1)$.

(d) The angular momentum $\mathbf{L}(1)$.

(e) The torque $\tau(1)$.

(f) The position $\mathbf{r}(1)$.

(g) The osculating plane equation at $\mathbf{r}(1)$.

(h) The tangential and normal components of acceleration $\mathbf{a}(1)$.