

**MATH 2551, Fall 2016**  
**Practice Exam 2, Chapter 14**

**Guideline:** Please read the following carefully.

Remember to show all your work; including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. One side of a letter sized sheet of paper for formula. Calculator is not allowed in this exam. You have 50 minutes.

**Problem 1.** Calculations.

(a) Find the directional derivative of  $f(x, y, z) = xy + yz + zx$  at  $P(1, -1, 1)$  in the direction of  $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

(b) Find the rate of change of  $f(x, y) = xe^y + ye^{-x}$  along the curve  $\mathbf{r}(t) = (\ln t)\mathbf{i} + t(\ln t)\mathbf{j}$ .

(c) Find  $\frac{\partial u}{\partial s}$  for  $u = x^2 - xy$ ,  $x = s \cos t$ ,  $y = t \sin s$ .

(d) Find  $\frac{dy}{dx}$  if  $x \cos(xy) + y \cos(x) = 2$ .

(e) Is  $\mathbf{F}(x, y) = (x + \sin y)\mathbf{i} + (x \cos y - 2y)\mathbf{j}$  a gradient of a function  $f(x, y)$ ? If yes, find the general form of  $f(x, y)$ .

(f) Set  $f(x, y) = \frac{x^2 - y^4}{x^3 - y^4}$ . Determine whether or not  $f$  has a limit at  $(1, 1)$ .

**Problem 2** Consider the function  $f(x, y, z) = \sqrt{x} + \sqrt{y} + \sqrt{z}$ .

(a) Find the equation for the tangent plane to the level surface  $f = 4$  at the point  $P(1, 4, 1)$ .

(b) Find the equation for the normal line to  $f = 4$  at  $P(1, 4, 1)$ .

(c) Use differentials to estimate  $f(0.9, 4.1, 1.1)$ .

**Problem 3.** Find the area of the largest rectangle with edges parallel to the coordinate axes that can be inscribed in the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$ .

**Problem 4** Find the absolute extreme values taken on  $f(x, y) = \frac{-2y}{x^2+y^2+1}$  on

the set  $D = \{(x, y) : x^2 + y^2 \leq 4\}$ .