MATH 2551, Fall 2016
Practice Final

Guideline: Please read the following carefully.

Remember to show all your work; including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. One sheet of paper (letter size) for formulas (one side) is allowed. Calculator is not allowed in this exam. Try to work this practice within 150 minutes.

Problem 1. This problem is about the function

$$f(x, y, z) = 3zy + 4xcos(z).$$

(a) Find the rate of change of the function f at $(1, 1, 0)$ in the direction from this point to the origin.

(b) Give an approximate value of $f(0.9, 1.2, 0.11)$

(c) The equation $f(x, y, z) = 4$ implicitly defines z as a function of (x, y), if we agree that $z = 0$ if $(x, y) = (1, 1)$. Find the numnerical values of the derivatives:

$\frac{\partial z}{\partial x}(1, 1)$ and $\frac{\partial z}{\partial y}(1, 1)$.

(d) Suppose $\mathbf{r}(t) = (x(t), y(t), z(t))$ is a parametric curve such that $\mathbf{r}(0) =$

$(1,1,0)$ and $\mathbf{r}'(0) = (3,2,1)$. Find the value of

$$\frac{d}{dt} f(\mathbf{r}(t))|_{t=0}.$$

Problem 2 (a) Find the value of a such that the field on the plane

$$\mathbf{F}(x,y) = (axy)\mathbf{i} + x^2\mathbf{j}$$

is conservative. Find a potential for the resulting field.

(b) Compute the line integral of the conservative field you found in part (a) over the curve $\mathbf{r}(t) = e^{t^2}\mathbf{i} + tcos(2\pi t)\mathbf{j}$, $0 \leq t \leq 1$.

Problem 3. Evaluate $I = \displaystyle\int_{C_R} dx + x^2 y dy$, where C_R is the triangle with vertices $(0,0)$, $(0,R)$, $(R,0)$ oriented counterclockwise.

Problem 4 Let S be the portion of the surface $x = 5 - y^2 - z^2$ in the half space $x \geq 1$, oriented so that the normal vector at $(5,0,0)$ is equal to $\mathbf{i}$. Let $\mathbf{F}(x,y,z) = -\mathbf{i} + \mathbf{j}$ (a constant vector field).

(a) Set up and evaluate the flux of $\mathbf{F}$ across S.

(b) Verify that $\mathbf{F} = \nabla \times \mathbf{G}$, where $\mathbf{G} = z\mathbf{j} - x\mathbf{k}$.

(c) Give an alternative calculation of the surface integral of part (a) by applying Stokes' theorem.

Problem 5 For each item, circle the correct answer or indicate if the statement is ture or false. Assume that the functions, fields and curves below are smooth.

(a) Let C be an arc from $(0,0)$ to $(2,1)$. According to the fundamental theorem for line integrals, $\int_C (y-1)dx + (x+2y)dy$ is equal to

(1) 2, (2) 1, (3) It depends on what C is.

(b) For every smooth function f, the integral $\int_0^1 \int_0^{2y^2+1} f(x,y)\, dxdy$ is equal to

(1) $\int_0^3 \int_0^{\sqrt{\frac{1}{2}(x-1)}} f(x,y)\, dydx,$

(2) $\int_1^3 \int_0^{\sqrt{\frac{1}{2}(x+1)}} f(x,y)\, dydx,$

(3) None of the above.

(c) If $\mathbf{F}$ is a field such that $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ where C is the unit circle, then $\mathbf{F}$ must be conservative.

(1) True, (2) False.

(d) If C is the boundary of a bounded domain D and C is oriented as in the statement of Green's theorem, then $\int_C x^2 y dx - y dy$ equals

(1) $\int\int_D (2xy - 1) dxdy$

(2) $\int\int_D (1 - x^2) dxdy$

(3) $\int\int_D (-x^2) dxdy$

(4) None of the above.

(e) If (a, b) is a critical point of a function f, and if

$$f_{xx}(a, b) = -2, \ \textit{and} \ f_{yy}(a, b) = 1,$$

then what can one say about (a, b)?

(1) Noting can be concluded from the given information.

(2) (a, b) is a local minimum of f

(3) (a, b) is a local maximum of f

(4) (a, b) is a saddle point of f

Problem 6 Consider the surface S that is the part of the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 3$.

(a) Give a parametirc representation of S. Make sure to explicitly describe or sketch the parametrization domain D.

(b) Find an equation of the tangent plane to S at the point $P(-1, 1, \sqrt{2})$.

(c) If the density function $\lambda(x, y, z)$ is equal to the distance to the xy-plane, find the total mass of the surface S.

Problem 7 Let E denote the portion of the solid ball of radius R centered at the origin in the first octant, and let

$$\mathbf{F} = (2x + y)\mathbf{i} + y^2\mathbf{j} + cos(xy)\mathbf{k}.$$

Applying the Divergence Theorem, compute the net flux of the field $\mathbf{F}$ across the boundary of E, oriented by the outward-pointing normal vectors.

Problem 8 Please complete the course survey. Your comments will help me to improve my teaching in the future. Thank you in advance.