

MATH 2551, Fall 2018
Practice Exam 2, Chapter 14

Guideline: Please read the following carefully.

Remember to show all your work; including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. One side of a letter sized sheet of paper for formula. Calculator is not allowed in this exam. You have 50 minutes.

Problem 1. Calculations.

(a) Find the directional derivative of $f(x, y, z) = xy + yz + zx$ at $P(1, -1, 1)$ in the direction of $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

(b) Find the rate of change of $f(x, y) = xe^y + ye^{-x}$ along the curve $\mathbf{r}(t) = (\ln t)\mathbf{i} + t(\ln t)\mathbf{j}$.

(c) Find $\frac{\partial u}{\partial s}$ for $u = x^2 - xy$, $x = s \cos t$, $y = t \sin s$.

(d) Find $\frac{dy}{dx}$ if $x \cos(xy) + y \cos(x) = 2$.

(e) Is $\mathbf{F}(x, y) = (x + \sin y)\mathbf{i} + (x \cos y - 2y)\mathbf{j}$ a gradient of a function $f(x, y)$? If yes, find the general form of $f(x, y)$.

(f) Set $f(x, y) = \frac{x^2 - y^4}{x^3 - y^4}$. Determine whether or not f has a limit at $(1, 1)$.

Problem 2 Consider the function $f(x, y, z) = \sqrt{x} + \sqrt{y} + \sqrt{z}$.

(a) Find the equation for the tangent plane to the level surface $f = 4$ at the point $P(1, 4, 1)$.

(b) Find the equation for the normal line to $f = 4$ at $P(1, 4, 1)$.

(c) Use differentials to estimate $f(0.9, 4.1, 1.1)$.

Problem 3. Find the area of the largest rectangle with edges parallel to the coordinate axes that can be inscribed in the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

Problem 4 Find the absolute extreme values taken on $f(x, y) = \frac{-2y}{x^2+y^2+1}$ on

the set $D = \{(x, y) : x^2 + y^2 \leq 4\}$.