Course: CS 1050C – Test 1 (FALL '03)

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Total Points : 10 + 10 + 10 = 30 plus extra credit problem = 5 pts.

Please explain all your answers.

Time: 50 minutes

Problem 1. (10pts) (a) Compute the gcd(2628, 738) using Euclid's gcd algorithm.

(b) Show that if $a^3|b^2$ then $a|b$, where a and b are positive integers.

Problem 2. (10 pts) Let $d = gcd(m, n)$, where m and n are positive integers.

(a) Prove or disprove: Suppose c is a common divisor of m and n. Then c divides d.

(b) We saw in class that d can be written as $mx + ny$ where x and y are integers. Can a positive integer smaller than d be written as $mx_1 + ny_1$ for some integers x_1 and y_1? (Recall that d divides both m and n). Explain your answer.

Problem 3. (10 pts) Let $\{a_n\}$, for $n = 0, 1, 2, \ldots$ be a sequence satisfying $a_n = 2a_{n-1} + 3a_{n-2}$, for $n \geq 3$. Given that $a_1 = a_2 = 1$, prove that $a_n = \frac{1}{2}(3^{n-1} - (-1)^n)$ for $n \geq 1$.

Extra Credit. (5 pts) The Fibonacci sequence $\{F_n\}$ is defined as follows. $F_0 = F_1 = 1$. For $n \geq 2$, F_n, the nth Fibonacci number satisfies the recurrence:

$$F_n = F_{n-1} + F_{n-2}.$$

Show that every pair of consecutive Fibonacci numbers is relatively prime. (Namely, show that $\gcd(F_n, F_{n-1}) = 1$ for $n \geq 1$.)