Course: CS 1050C - QUIZ 2 (FALL '03)

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Time: 20 minutes Total Points : 5+5+5 = 15 pts.

Please write your name and explain all your answers.

Name: Prasad Tetali Date: Oct. 24th, 2003.

Problem 1. In a group of 1100 people must at least four people have the same birthday? How about at least five people having the same birthday?

Yes to the first, and No to the second. Using the generalized pigeon-hole principle, we may observe that no matter how we distribute the birthdays over the 365 days, there would be at least one day with at least four birthdays, since $3 \times 365 < 1100$. (note that we have room to spare, even if we count 366 as the no. of days in a year.)

To guarantee at least five fall on the same day, we would need more than 4×365 birthdays. We could very easily distribute the 1100 birthdays over 365 days with each day taking four or less birthdays.

Problem 2. Answer in True or False:

(a) If $f: A \to B$ is onto then $|A| \ge |B|$.

True : onto guarantees that for each $b \in B$, there exists an $a \in A$ so that f(a) = b (and since f is a function, there can not be two b's for the same a), and so $|A| \ge |B|$.

(b) Any subset of the rationals is countable.

True: The set of rationals is countable, and any subset of countable is countable.

(c) The power set $\mathcal{P}(S)$ of a countable set S is countable.

False : mentioned in class that the power set of a set is not necessarily countable, even if the set is countable. (Also mentioned that exercise 27 in Sec. 7.6 covers this: it implies in particular that \mathbf{Z} and $\mathcal{P}(\mathbf{Z})$ do not have the same cardinality.)

(d) The function $f : \mathbf{R}^+ \to \mathbf{R}^+$ defined as $f(x) = x^3$ is one-to-one and onto. (Here R^+ is the set of positive reals.)

True : one-to-one, since $x_1 \neq x_2$ implies that $x_1^3 \neq x_2^3$. onto because y has a cube root, for each $y \in \mathbf{R}^+$.

(e) Showing that $f : X \to Y$ is a bijection means showing that for each $y \in Y$, the equation f(x) = y has a unique solution in X.

True: the fact that there is a solution, shows that f is *onto*, and the fact that the solution is *unique* shows that it is one-to-one: $f(x_1) = f(x_2) = y$ implies $x_1 = x_2$.

Problem 3. Does multiplication by 2 define a bijection from \mathbf{R} to \mathbf{R} ? how about from \mathbf{Z} to \mathbf{Z} ?

Yes to the first, and No the second: f(x) = 2x can easily be verified to be one-to-one and onto when $f : \mathbf{R} \to \mathbf{R}$.

But if $f : \mathbf{Z} \to \mathbf{Z}$, it is not onto. For consider an odd integer $m \in \mathbf{Z}$, then there is no integer n such that f(n) = m, since f(n) = 2n.