Course: CS 4803/8803 (Spring'08) – Homework 2

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Office Hours: **Wed. Fri. 4:30-5:30pm, and by appointment**

Due: next Wednesday

Problem 1. [Problem A.17] from the Appendix of Ron Roth's book.

Problem 2. Show that the number of distinct generator matrices of a linear $[n, k, d]$ code over the finite field $F = GF(2)$ is $\prod_{i=0}^{k-1}(2^k - 2^i)$.

Hint. First show that the sought number equals the number of $k \times k$ nonsingular matrices over F.

Problem 3. [Problem 2.4] Let $(a_1 a_2 \cdots a_k)$ be a nonzero vector over the finite field $F = GF(q)$ and consider the mapping $f : F^k \to F$ defined by $f(x_1, x_2, \cdots, x_k) = \sum_{i=1}^{k} a_i x_i$. Show that each element of F is the image under f of exactly q^{k-1} vectors in F^k. (If you are not comfortable with $GF(q)$ for general q yet, just solve for $q = 2$.)

Problem 4. [Problem 2.5] Show that in every linear code $F = GF(2)$, either all codewords have even Hamming weight or exactly half of the codewords have even Hamming weight.

Hint. Let G be the $n \times k$ generator matrix; i.e. the code is $\mathcal{C} = \{Gx : x \in F^k\}$. See when Problem 3 above can be applied to the mapping $F^k \to F$ that is defined by $\mathbf{x} \to (11 \cdots 1)G\mathbf{x}$.

Problem 5. [Problem 2.6] Let $\mathcal{C}$ be a linear $[n, k, d]$ code over $F = GF(q)$ and let T be a $q^k \times n$ array whose rows are the codewords of $\mathcal{C}$. Show that each element of F appears in every nonzero column in T exactly q^{k-1} times.

Hint. Use Problem 3 above. Will discuss in class.

Problem 6. [Problem 2.8] Let G be a generator matrix of a linear code $\mathcal{C} \neq \{\mathbf{0}\}$ over a field F. Show that the minimum distance of $\mathcal{C}$ is the largest integer d such that every $(n - d + 1) \times k$ sub-matrix of G has rank k.