Math 2406B (Fall 08) – Homework 2 (Due: Friday, Sept. 12th)

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1. Let A, B, and C be sets, and let $f : A \to B$ and $g : B \to C$ be two functions. Recall the definition of *composition* $g \circ f$ of functions f and g. This is a function $g \circ f : A \to C$ defined as $g \circ f(a) = g(f(a))$, for every $a \in A$.

Prove that if f and g are onto (also known as, a surjection) then $g \circ f$ is also onto.

2. Let $V = \{(a_1, a_2) : a_1, a_2 \in \mathbf{R}\}$. Define *vector addition* and *scalar multiplication* as follows: for $A = (a_1, a_2)$, $B = (b_1, b_2)$, and $c \in \mathbf{R}$,

$$A + B = (a_1 + b_1, a_2 - b_2), \qquad \text{and} \qquad cA = (ca_1, ca_2)\,.$$

Does the above yield a vector space (a.k.a. linear space). If not, determine which of the axioms fail to hold.

3. A real-valued function f defined on the real line is called an **even function** if $f(-x) = f(x)$ for each $x \in \mathbf{R}$. Prove that the set of even functions, with the usual definitions of addition and scalar multiplication for function spaces, forms a vector space.

- Section 3.5 : Exercises 9, 22(a), 25 (a).

- Section 3.10 : Exercise 8, 23 (a)

- **Optional Problems** :
  Section 3.5 : 12, 17
  Section 3.10 : 20, 25.

Advisory Remarks: Try to make proofs succinct, clear and complete. The grader or I need to understand your logic, so please try to communicate your explanation well.

The book might have answers at the end, but you need to explain (or prove) and not simply report the answer.