Math 2406B (Fall 08) – Homework 6 (Due: Friday, Nov. 14th)

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1. Let V and W be finite-dimensional linear spaces and $T : V \to W$ be an isomorphism. Let V_0 be a subspace of V.
(a) Prove that $T(V_0)$ is a subspace of W.
(b) Prove that $\dim(V_0) = \dim(T(V_0))$.

2. Repeat the example done in class, illustrating the commuting diagram, with $p(x) = 1 + x + 2x^2 + x^3$. To restate: let $T : P_3(\mathbf{R}) \to \mathbf{P_2}(\mathbf{R})$ be the differentiation operator from the set of polynomials (over the reals) of degree at most 3 to that of degree at most 2. Take the standard (ordered) bases $\mathcal{B}_1$ and $\mathcal{B}_2$ for $P_3(\mathbf{R})$ and for $P_2(\mathbf{R})$, respectively. Let $A = m(T)$ be the matrix corresponding to T with respect to these bases. Also recall that $L_A : \mathbf{R^n} \to \mathbf{R^m}$ is the linear transformation corresponding to A, and $\phi_{\mathcal{B}} : V \to R^n$ is the vector representing an element of linear space V using the basis elements from $\mathcal{B}$.

Show that $L_A \phi_{\mathcal{B}_1}(p(x)) = \phi_{B_2} T(p(x))$.

3. Let A be an $m \times n$ matrix. If P is an $m \times m$ invertible matrix, then show that rank(PA) = rank(A).

4. Let V be a linear space, and suppose that T and S are linear transformations from V to V, such that S is onto and the null spaces of T and S are finite-dimensional. Then the null space of $T \circ S$ is finite-dimensional, and

$$\dim(N(T \circ S)) = \dim(N(T)) + \dim(N(S)) \ .$$

(*Hint*: Let $\{u_1, u_2, \ldots, u_n\}$ be a basis for $N(T)$. Let $\{v_1, v_2, \ldots, v_m\}$ be a basis for $N(S)$. Let $\{w_1, w_2, \ldots, w_n\}$ be such that $S(w_j) = u_j$, for $j = 1, 2, \ldots, n$. (Pause: how do we know w_j exist?) Show that the v_i and the w_j are distinct and that together they form a basis for $N(T \circ S)$.)

• **Section 5.20** : Exercises 3(a, b), 4(c)

Advisory Remarks: Try to make proofs succinct, clear and complete. The grader or I need to understand your logic, so please try to communicate your explanation well.

The book might have answers at the end, but you need to explain (or prove) and not simply report the answer.