

④. Solution (continued):

To show $\{v_i\}_{i=1}^m$ and $\{w_j\}_{j=1}^n$: indep.

Suppose that $\sum_i c_i v_i + \sum_j c_j w_j = 0 \rightarrow \textcircled{1}$

Then applying S ~~to~~ to $\textcircled{1}$, and using $S(v_i) = 0$,

we get $\sum_j c_j S(w_j) = 0$

$\Rightarrow \sum_j c_j u_j = 0$, since $S(w_j) = u_j$.

$\Rightarrow c_j = 0, \forall j$, since u_j : independent.