MATH 3012 Applied Combinatorics (Fall'07) – Quiz 1

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Time: 30 minutes

1. What is the coefficient of the term containing x^3y^6 in the expansion of $(2x-3y)^9$?

Soln. It is $\binom{9}{3}2^3(-3)^6$

2. Describe two combinatorial structures whose counting function is the sequence of Catalan numbers.

Soln. (i) The number of (planar) triangulations of a regular polygon with $n+2$ sides. (ii) The number of binary sequences with n 1's and n 0's, where the number of 1's is at least as large as the number of 0's as we count from left to right.

3. (a) How many cyclic permutations of length n are there?
(b) How many binary strings of length 10 are there with four 1's?

Soln. (a) The question is a bit vague – one answer is $(n-1)!$ accounting for the fact that each of the n cyclic rotations of a given linear permutation is treated as the same. The other answer is n, if one interprets the question as taking a permutation and simply rotating it n times. (The instructor meant the former, but will give credit for the latter as well.)

4. There are 24 men and 8 women in a company. In how many ways can one choose a committee consisting of six people, if at least two women must be chosen.

Soln. The total number of committees with 6 out of 32 people minus the number of 6-people committees with *fewer than* 2 women:

$$\binom{32}{6} - \binom{24}{6}\binom{8}{0} - \binom{24}{5}\binom{8}{1} = \sum_{i=2}^{6}\binom{8}{i}\binom{24}{6-i},$$

the latter being interpretable as a direct count of numbers of committees with precisely i women (and $6-i$ men) in the committee, for $2 \le i \le 6$.