Math 4022 (Fall 08) – TEST 1 (September 22, 2008)

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Answer the first THREE in class; bring the fourth to class on Wednesday
Please EXPLAIN All Answers. No Calculators Allowed

Time : 50 minutes   Total Points : $10 + 10 + 5 + 10 = 35$

1. (5+5 points) (a) Provide a 3-regular graph with no perfect matching.

Solution.

(b) Find a *maximal* matching which is *not* a maximum matching in the Petersen graph:

Solution.

2. (5+5 points) (a) Find the labeled tree which corresponds to the Prüfer code $(3, 3, 4, 4)$.

Solution.

(b) Use the matrix-tree theorem to find the number of spanning trees of the following graph:

Solution.

3. (5 points) Prove that every *bipartite* graph has a matching of size at least $|E(G)|/\Delta(G)$, where $|E(G)|$ is the number of edges in G, and $\Delta(G)$ is the maximum degree in G.

(**Bonus 5 Points**). Conclude that every subgraph of $K_{n,n}$ with more than $(k-1)n$ edges has a matching of size at least k.

Solution.

4. (10 points) (The Politically Incorrect Dating Problem.) Show that in a group of m boys and n girls there exist some t boys for whom dates can be found if and only if any subset of the boys (k of them, say) between them know at least $k + t - m$ of the girls. (We are assuming a date is arranged only if boy knows girl.)

Hint 1: Think of the case of $t = m$ first.

Hint 2: Invite additional $m - t$ "very popular" girls who are known to all the boys. Show that at least t boys can find dates in the original situation if and only if *all* the boys can find dates in the new situation...

Solution.

Extra Space