Math 4022 (Fall 08) – Test 2 (Friday, Oct. 31st)

Instructor : Prasad Tetali, office: Skiles 234, ph: 404-894-9238
Webpage: http://www.math.gatech.edu/∼tetali **email**: tetali@math.gatech.edu

Solve the FIRST THREE now, and bring the REST ON MONDAY to class.

NAME: ____

Advisory Remarks: Try to make proofs succinct, clear and complete. The grader or I need to understand your logic, so please try to communicate your explanation well.

1. (12 points) (a) What is the vertex connectivity of the discrete n-cube Q_n?

(b) Does the Petersen graph have an ear decomposition? Explain why or why not.

(c) State the Fan Lemma for k-connected graphs.

(d) Given an s-t network, define *mincut* and *maxflow*.

2. (5+5 points) (a) Give an example graph which satisfies all three of the following conditions: vertex connectivity 3, edge connectivity 4 and minimum degree 5.

(b) Find the maximum flow in the following network.

3. (8 points) (a) Use Menger's theorem ($\kappa(x, y) = \lambda(x, y)$ when $xy \notin E(G)$) to prove König-Egerváry theorem ($\alpha'(G) = \beta(G)$, when G is bipartite.)

Take-home Part of Test 2

4 (10 points). Let G be a weighted undirected graph, with a nonnegative real as the weight on every edge. Let the *value* (denoted *val*) of a spanning tree be the minimum weight of its edges. Let the *cap* of an edge cut $[S, \bar{S}]$ be the maximum weight of its edges. (For a subset S of vertices, $\bar{S}$ denotes the complement of S.)

Prove that the maximum value of a spanning tree of G equals the minimum cap of an edge cut in G. That is, prove that

$$\max_{T\text{:sp. tree}} val(T) = \min_{[S:\bar{S}]} cap([S : \bar{S}])\,.$$

Hint: Proving max at most min *is easy, as has been the case so far, with other theorems. So try that direction first.*

5 (10 points). Find the maximum flow in the following network, starting with a zero flow. Show that your flow is optimal by finding a minimu cut of appropriate capacity.