MATH 4022 (Intro to Graph Theory) Homework 1

Due: Wednesday, Sept. 7, 2016 (in class)

- **Instructor:** Prasad Tetali, tetali-at-math-dot-gatech-dot-edu; 404-894-9238 (o)
- **Class Location and Time:** Skiles 268, MW 3:05–4:25pm
- **Office hours:** Skiles 118B, Monday 4:30-5:30, Tuesday, Friday 2:00–3:00pm

Most of the following problems are from the textbook by Doug West.

1. Let G_n be the graph whose vertices are the permutations of $\{1, 2, \ldots, n\}$, with two permutations adjacent if they differ by interchanging a pair of adjacent entries. Prove that G_n is connected.

2. Determine the values of m and n such the complete bipartite graph $K_{m,n}$ is Eulerian (i.e., contains an Eulerian circuit).

3. Prove that the Ramsey number $R(4, 3) \leq 10$. (*Hint.* You may use that $R(3, 3) = 6$.)

4. The *Petersen graph* is the simple graph whose vertices are the 2-element subsets of a 5-element set and whose edges are the pairs of disjoint 2-element subsets.

(a) Draw the Petersen graph.

(b) Determine whether it is bipartite.

(c) What is the size of the largest independent set in this graph.

(d) Let C be a subset of vertices with the property that every edge of the graph has at least one endpoint in C. What is the size of the smallest (in cardinality) such a C for the Petersen graph?

5. Using the Prüfer correspondence, count the following sets of trees with vertex set $[n]$

a) the number of trees with 2 leaves.

b) the number of trees with $n - 2$ leaves.

6. Let T, T' be two spanning trees of a connected graph G. For $e \in E(T) - E(T')$, prove that there is an edge $e' \in E(T') - E(T)$ such that $T' + e - e'$ and $T - e + e'$ are both spanning trees of G.

7. Use the matrix-tree theorem to prove Cayley's formula.
(*Hint.* Apply row-reduction operations to make most elements zero, so that the determinant becomes very easy to compute.)