MATH 4022 (Intro to Graph Theory) Homework 4

Due: Monday, November 14, 2016 (in class)

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- **Office hours:** Skiles 118B, Monday 4:30-5:30, Tuesday, Friday 2:00–3:00pm

Most of the following problems are from the textbook by Doug West.

1. (Problem 4.1.5) Let G be a connected graph with at least three vertices. Form G' from G by adding an edge with endpoints x, y whenever the distance in G between them $d_G(x, y) = 2$. Prove that G' is 2-connected.

2. (Problem 4.1.10) Find the smallest 3-regular graph with connectivity 1. (Note that it must have a cut-edge.)

3. Problem 4.1.14) Let G be a connected graph in which for every edge e, there are cycles C_1 and C_2 containing e whose only common edge is e. Prove that G is 3-edge-connected. Use this to show that the Petersen graph is 3-edge-connected.

4. (Problem 4.2.4) Prove or disprove: If P is u, v-path in a 2-connected graph G, then there is a u, v-path Q internally disjoint from P.

5. (Problem 4.2.8) Prove that a simple graph G is 2-connected, if for every ordered triple (x, y, z) of vertices, G has an x, z path through y. (Use the Fan Lemma.)

6. (Problem 4.2.12) Use Menger's theorem to prove that $\kappa(G) = \kappa'(G)$, when the graph G is 3-regular.

7. (Problem 4.2.13) Let G be 2-edge-connected. Define a relation R on the edge set $E(G)$ by $(e, f) \in R$ (that is, e and f are related) if $e = f$ or if $G - e - f$ is disconnected.

a) Prove that $(e, f) \in R$ if and only if e, f belong to the same cycles.

b) Prove that R is an equivalence relation on $E(G)$.