MATH 4022 (Intro to Graph Theory) Homework 5

Due: NO NEED TO TURN IN

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- **Office hours:** Skiles 118B, Monday 4:30-5:30, Tuesday, Friday 2:00–3:00pm

Most of the following problems are from the textbook by Doug West.

1. What is the chromatic number and chromatic index of the Petersen graph?

2. (Problem 5.1.20) Let G be a graph whose odd cycles are *pair-wise intersecting*, meaning every two odd cycles have a vertex in common. Show that $\chi(G) \leq 5$.

3. (Problem 5.1.21) Suppose that every edge of a graph G appears in at most one cycle. Show that every block of the graph is an ege, a cycle or an isolated vertex. Use this to show that $\chi(G) \leq 3$.

4. (Problem 5.1.28) Consider a traffic signal controlled by two switches, each of which can be set in one of n positions. For each setting of the switches, the signal shows one of its n colors. Whenever the setting of *both* switches changes, the color changes. Prove that the color shown is determined by the position of one of the switches. Interpret this in terms of the chromatic number of some graph.

5. (Problem 5.1.38) Suppose G is bipartite. Prove that $\chi(\bar{G}) = \omega(\bar{G})$, where $\bar{G}$ is the complement of G.

6. (Problem 5.1.41) Prove that $\chi(G) + \chi(\bar{G}) \leq n + 1$, where n is the number of vertices. (*Hint* : Use induction on n.)

7. (Problem 5.2.21) The Turán graph $T_{n,r}$ is the complete r-partite graph with b partite sets of size $a + 1$ and $r - b$ partite sets of size a, where $a = \lfloor n/r \rfloor$ and $b = n - ra$.

a) Prove that the number of edges in $T_{n,r}$ is $e(T_{n,r}) = (1 - 1/r)n^2/2 - b(r - b)/2r$.

b) Since the number of edges is an integer, part a) implies that $e(T_{n,r}) \leq \lfloor (1 - 1/r)n^2/2 \rfloor$. Determine when strict inequality occurs.

8. (Problem 5.3.4) Prove that the chromatic polynomial $\chi(C_n; k)$ of an n-cycle with k colors is $(k - 1)^n + (-1)^n(k - 1)$.