

MATH 4022 (Intro to Graph Theory) Quiz 2

October 24, 2016 (in class, closed book, closed notes); duration: 40 minutes

• Name of Student:

Score:_____/30

Please provide a proof whenever appropriate; do not simply write an answer without due explanation or justification. Total score: 30 points.

1. (4 pts) Is the inequality $\beta(G) \geq \alpha'(G)$ always true (whether G is bipartite or not)?

Answer.

2. (4 pts) State the Turán bound for the independence number of a graph?

Answer.

3. (4 pts) Suppose a graph has no triangles. Then why is the independence number at least the maximum degree in the graph?

Answer.

4. (4 pts) Petersen showed that every 3-regular graph with no cut-edge has a perfect matching (also known as a *1-factor*). Show an example that shows the assumption of “no cut-edge” is necessary. (That is, show a 3-regular graph which has a cut-edge and no perfect matching.)

Answer.

5. (4 pts) What is the difference between a *maximal* matching and a *maximum* matching?

Answer.

6. (4 pts) Suppose a tree has a perfect matching. Show that the number of odd components $o(G - \{v\}) = 1$, when we remove any vertex v in the tree.

Answer.

7. (4+2=6 pts) Color each edge of K_6 Red or Blue with equal probability, and independently over the edges. Compute the expected number of monochromatic cliques of size 4. (Recall that a clique is monochromatic if all its edges get the same color.) What conclusion can you make?

Answer.