MATH 4022 Intro to Graph Theory (Fall'07) – TEST 1

Instructor : Prasad Tetali, office: Skiles 234, email: tetali@math.gatech.edu Time: 1 hour 25 minutes Total Score: 50 pts

Solve the first four in class. Bring the solution to the fifth to class on Monday.

Name : _____

1 (10pts). Find a maximum matching, minimum vertex cover, a largest independent set and a minimum edge cover in the Petersen graph. Please justify your answer.

2 (10pts). Count the number of spanning trees in the complete bipartite graph $K_{2,n}$.

3 (5+5pts). (a) Show that if a tree G has a perfect matching, then o(G - v) = 1, for every vertex v in G, where o(G - v) refers to the number of components of odd size in G - v (which is the graph G with v and its incident edges removed.)

(b) If the Prüfer code of a spanning tree T of K_8 is (4, 1, 4, 3, 1, 5), what is T? (Note that this has nothing to do with Part (a).)

4 (10pts). Prove that a graph G is bipartite if and only if every subgraph H of G has an independent set consisting of at least half of V(H).

5 (10pts). Let M be a maximal matching and L a minimal edge cover in a graph with no isolated vertices. Prove the following statements.

(a) M is a maximum matching if and only if M is contained in a minimum edge cover.

(b) L is a minimum edge cover if and only if L contains a maximum matching.